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The Augmented ACD Models: High Frequency Modelling and Applications to BVMT Stocks

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Abstract

We propose in this paper, a new work to model the durations between successive transactions of the Stock Exchange of Tunis (BVMT). For this purpose, the autoregressive approach of the ACD model will be extended to the class of augmented ACD models to model the data that arrive at irregularly spaced intervals in time called high-frequency data or Ultra-high frequency data. The choice of the interval remains crucial since the daily exchanges are too small.

Keywords: Financial time transaction; autoregressive conditional duration models; augmented ACD models; aggregation.

1. Introduction

In Tunisia, since October 1996, an electronic quotation system (super CAC) has been put in place to replace the open outcry system. It is managed by a central computer allowing the confrontation of buy and sell orders, and thus the determination of the equilibrium price. This system has been modernized once again by a new version of quotation developed by "NYSE/EURONEXT V900" and the quotation schedule in October 2008 went from 2h 40m to 5h 10m. This new quotation system has favored the recording of high frequency data which is sometimes called "ultra-high frequency data."

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Producing accurate predictions is a priority for financial time series models. The Recent theoretical and empirical research in econometrics and statistics has shown a growing interest in modeling high frequency data. ultimate limiting case is reached when all individual events are recorded. Engle (2000) calls this limit frequency "ultra-high frequency." The examination of ultra-high frequency data is an extremely active area of research, particularly in financial economics and econometrics. It is an obvious consequence of the availability of intraday data bases with detailed information on the entire trading process involving, in the limited case, all individual trades and orders in a financial market.

The authors [2] link the extensive literature on GARCH models [3] to the econometric literature on duration data [4,5]. They propose point process modelling of the overdispersion and persistence of inter-transaction arrival times typically observed in high-frequency financial data (stock market transactions [2,6] and exchange rate transactions [7]).

On the residuals of the ACD model, researchers in [2] found evidence supporting non-linear effects of recent durations on the conditional mean that appear to be smaller than those predicted by the linear specification for very long and very short durations.

Durations have a plausible role in price discovery in markets characterized by the presence of traders with different levels of information about the underlying value of the assets being traded. In the financial literature, for the study and development of time series models of inter-transaction arrival times, the models of [8,9] have provided theoretical justifications for studying such series.

A key property of transaction data is irregular spacing over time. The question of how this salient feature should be treated in an econometric model is one of the most controversial issues in high-frequency data econometrics. It is clear that researchers can get around this problem by aggregating the data into fixed (discrete) intervals. However, such a procedure is naturally accompanied by a loss of information and raises the question of an optimal level of aggregation. Moreover, it is not clear what kind of bias is induced by the choice of a discrete sampling scheme. In this context, two major aspects are particularly important. First, the time interval between subsequent events itself has information content and is a valuable economic variable that serves as a measure of business activity and can affect price and volume behavior. By aggregating the data to equidistant intervals, this information is discarded. The second aspect is more technical, but no less crucial. Ignoring irregular data spacing in an econometric model can lead to significant misspecification. As illustrated in [10], there are two effects that need to be taken into account in this context: the effects of sampling discretization and random sampling. These effects are associated with the implications when irregularly spaced data are sampled at discrete and equidistant time intervals. The latter is related to the additional effect that the randomness of the sampling intervals has when a continuous-time model is estimated.

The availability of high-frequency recorded financial data has inspired a field of research that in recent decades has emerged the main areas of econometrics and statistics. The growing popularity of high-frequency econometrics is triggered by the progression of technology in the trading system, trade recording and of course the significant growth in daily trading, optimality in order execution and liquidity dynamics. Technological advances and the increasing dominance of electronic trading allow for high frequency market activity to be recorded with great accuracy, allowing for a good understanding of the data. The case of an informational limit is reached when all market events in message form are recorded.

The objective of this paper is to provide an overview of most approaches in high frequency econometrics. The main objective is to discuss the implementation of high frequency data properties and to present applications on the autoregressive conditional duration models ACD and Augmented ACD model. The important task in high frequency data modeling is to appropriately capture the dynamics of the data. In this context, modeling the conditional mean as an autoregressive process plays an important role in the literature. Our methodology is based on the application of these new econometric models with a non-negative error term component. [1] paper can be seen as the starting point of a rapidly growing body of research in high frequency financial econometrics.

In this paper, we present a work of methodological interest by which we begin with a reprocessing of the data that is essential to this type of study, since each day we have a set of transactions that are carried out at different times of the day and therefore, they must be associated with each trading day. In addition, in order to improve the readability and fluidity of the analyses and given the very large amount of data, we have opted for a representation by tables and graphs of the different results. This new database constitutes a new area of further research since it allows us to understand exactly and in real time, the behavior of the participants as well as the structure of the Tunisian market governed by the orders in order to enrich the knowledge still too rare on this structure that is why, they seem to us that the main interest of this research is of empirical order.

Our paper is organized as follows, a first section devoted to the statistical description of the data and their treatment, the second section focuses on the production process of which we present a range of different ACD models and augmented ACD models that are presented in terms of a random coefficient polynomial. The estimation of these different models is in the third section while, the estimation of the aggregated data in 5 and 30 minutes is in the fourth section. The conclusion is in the fifth section.

2. Descriptive statistics of raw data

The functional characteristics of the time series of transaction data or "ultra-high frequency (UHF) data" known as "tick-by-tick data" include the non-synchronization of the distributions of observations over the unit of time, discrete transaction prices, the appearance of a large number of transactions in the same second, and the existence of intra-day seasonality, i.e., these transactions reveal a daily periodic cycle. These are time series composed by the characteristics of the trading events to which the exact time of their appearance was assigned. These observations are recorded asynchronously in units of time and have certain quality characteristics that do not occur at low frequencies.

2.1. Data processing

The sampling of the series will be for each (x) transaction and by hypothesis, each transaction brings the same amount of information to the market. The first treatment concerns the first two columns that must be converted to the POSIXlt format, that is, we will have a single column of the form yyyy-mm-dd hh:mm:ss (year-monthday-hour-minute-second). The continuous trading session starts with a pre-opening from 9am to 10am in which the orders are entered without any transaction at a fixing price at the opening for all the stocks we have eliminated since it is a price that is adjusted before the session takes place. Thus, our trading session starts at 10am and ends at 2pm.

The time taken to complete two successive transactions is called trade duration and is the natural measure of trade intensity. The importance of time intervals between transactions in the trading process is well discussed in market microstructure theory, [8,11,12,13]. Since any trade reflects the demand for liquidity, the duration between trades is then associated with the demand for liquidity. We found the existence of several transactions made at the same time and as in [1,2,14] we eliminated the zero durations and the negative durations that arise from the difference between the opening date of day d and the closing date of day $d-1$ as shown in **table (1)** for the day of **02/01/2014** of the two stocks (durations are in seconds).

Table 1: Reprocessing of the two stocks Data

POSIXlt format	Adwya	POSIXlt format	Uib
Time	Durations	Time	Durations
2014-01-02 10:03:01	181	2014-01-02 12:31:29	9089
2014-01-02 10:03:21	20	2014-01-02 12:31:59	30
2014-01-02 10:47:25	2644	2014-01-02 13:57:46	5207
2014-01-02 10:47:52	27		
2014-01-02 10:57:28	576		
2014-01-02 11:17:09	1181		
2014-01-02 13:20:09	7380		
2014-01-02 13:33:28	799		

2.2. Descriptive statistics of financial durations

Table (2) shows the descriptive statistics of the financial durations of the two securities. As can be seen, we have a reduction in the observations of the two securities, an average of transactions of 7 minutes for the Adwya security and about 11 minutes for Uib, this is a waiting time that is too long, the authors in [12] qualify it as the arrival of bad news. By analyzing the characteristics of the distribution of the duration, we conclude the over-dispersion of the market, a coefficient of variation higher than the unit which implies a great dynamic of the series in question.

Table 2: Descriptive statistics of the financial durations

	Adwya	Uib
Nb. Obs	3589	2313
C.V	2.47	2.26
Min	1	1
Mean	427	650.9
Max	12329	14077
Q(10)	916.34	409.17
Q(100)	5926.4	1880.4

Nb. Obs: number of observations. C.V: coefficient of variation. Durations are in seconds. Q(k) is the statistic of Ljung. Box of the autocorrelation of order k.

The Ljung-Box statistics (Q(k)) test the null hypothesis that there is no auto-correlation for the different lags. Clearly, this hypothesis is rejected for both stocks against a too high auto-correlation of durations which indicates a strong auto-correlation of durations. The graphs of the auto-correlation functions of the durations of the two titles **(figure 1)** take a long time to decrease approximately at a hyperbolic speed which generally characterizes the phenomena of long memory or persistence of the process. We also notice, high values for low orders of auto-correlations which indicates well, strong clustering phenomena of transactions.

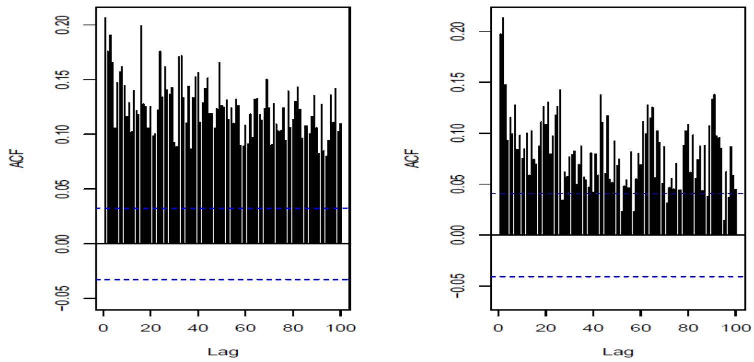


Figure 1: Autocorrelation function of transaction durations

The dynamics of the durations process and the clustering effect of the transaction activity are summarized in **figure (2)** of the first 1000 observations. We observe clusters in the time of financial durations such that long (short) durations are followed by long (short) durations, which suggests a positive dependence between the duration series that is well confirmed by the graphs in **figure (2)**.

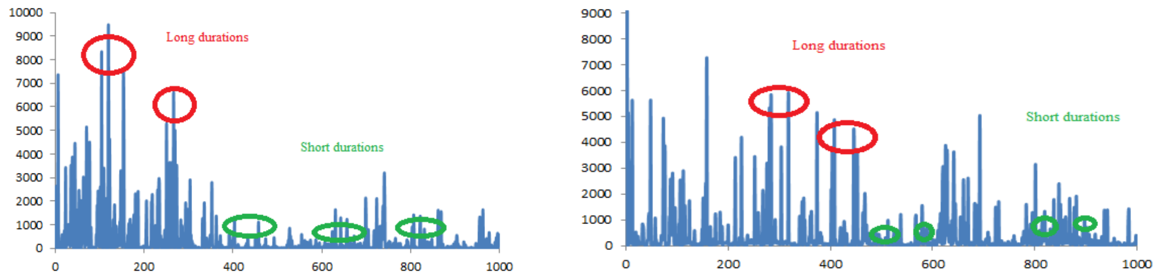


Figure 2: Evolution of the first 1000 durations. Adwya (right) and Uib (left)

2.3. Descriptive statistics of seasonally adjusted durations

The existence of strong autocorrelation in durations may result from a noon seasonality of transactional activity. With this in mind, the intraday seasonality cycles are estimated by a cubic spline with nodes at each hour of the day between 10:00 a.m. and 2:00 p.m. The authors of [2] proposed to estimate this intraday component by a cubic spline while Engle [1] estimates it by a linear spline with almost the same result for both techniques, then this intraday effect is removed from the data. To obtain the seasonally adjusted durations, we divide the duration by the corresponding seasonal factor, assuming that it is the same for all days of the week

$$\tilde{x}_{t_i} = \frac{x_{t_i}}{\phi_j(\tau_{t_i})} \qquad (1)$$

where x_{t_i} is the duration between event t_i and t_{i-1}, $\tilde{x}_{t_i}$is the adjusted duration (seasonally adjusted duration), and $\phi_j(\tau_{t_i})$ is the diurnally adjusted time of arrival of event (transaction) t_i at time i in day j. In the case of a cubic spline, we have

$$\varphi\left(t_{\{i\}}\right) = \xi^{3}_{\{k,3\}\left(t_{\{i\}}-t_{\{k\}}\right)} + \xi^{2}_{\{k,2\}\left(t_{\{i\}}-t_{\{k\}}\right)} + \xi_{\{k,1\}\left(t_{\{i\}}-t_{\{k\}}\right)} + \xi_{\{k,0\}}, \qquad (2)$$

For $t_k \leq t_i \leq t_{k+1}$ where $k = 0, \dots, k_0$, ($\xi_{\{k,j\}}$ is a real parameter), t_k is the value of the kth node at the transaction time $t_{\{i\}}$.

Table 3: Seasonally adjusted durations

	Adwya	Uib
Min	0.002	0.001
Median	0.14	0.19
Mean	1	1
Max	27.84	18.34
C.V	2.4	2.04
Q(20)	1702.7	650.51

Q(k) is the Lyung statistic- Box to 20 lags.

In general, it is a polynomial curve (in pieces) of degree d whose first $d-1$ derivatives are continuous and the points of discontinuity of the d'th derivative are called the nodes. We have chosen the nodes of one hour, i.e., 10h, 11h, 12h, 13h and 14h with a non-monotonous dynamic of the seasonal factor between the two nodes. **Table (3)** shows the descriptive statistics for seasonally adjusted durations.

Both charts exhibit a slight inverse U-shape and unequivocally reveal that durations are subject to daily seasonality **(figure 3)**.

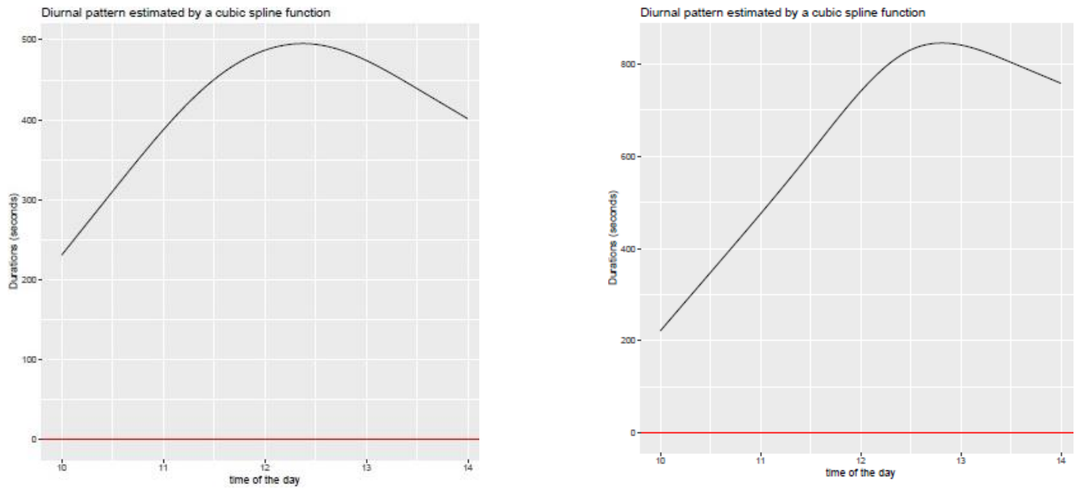


Figure 3: Diurnal pattern estimated by cubic spline function, Adwya (left) and Uib (right)

The durations between trades are clearly short after the opening of the session and with less movement before the close of the session than in the middle of the session. The magnitude of trading activity between 11:30am and 12:30pm is visibly low and this is due to the effect of investor fatigue which tends to rest especially at lunchtime [1,2,15,16]. Note, that this effect is less apparent for the Uib title, since the hump of the Adwya title graph is less sharp. Similarly, the market activity at the opening of the session is too intense because investors adjust the information of the previous night, while this activity at the closing can be explained by some investors who try to close an open position.

3. The duration processes

The implementation of an autoregressive structure at the point process level will be performed by an autoregressive process in terms of duration. While such a process is specified via independently and identically distributed intervals and very many types of point processes allow dynamics within intervals of successive events $\{x_{\{i\}}\}$, $i = 1, \ldots, n$. Thus, the class of what is called Wold process is obtained when the process $\{x_{\{i\}}\}$, $i = 1, \ldots, n$ forms a Markov chain whose distribution of $x_{\{i+1\}}$ having $x_{\{i\}}$, $x_{\{i-1\}}$, depends only on $x_{\{i\}}$. In fact, the AR (1) process for inter-event durations can be a good example of the Wold process.

By transforming the Markovian structure, several point dynamic processes are obtained. Thus, a high order dynamic of the duration process allows to obtain the so-called class of autoregressive duration processes. This process is characterized by an intensity function containing an autoregressive component that depends on past durations. This class of duration processes is proposed by [1,2,7] and thus, the autoregressive duration model is the best type of model that can be used for point specifications of financial processes and is the most widely used in the recent financial econometric literature.

However, durational models have major drawbacks. These models are not easily extended to a multivariate framework since the individual process does not occur in a time-synchronous manner which results in the non-existence of joint variables that can be used to couple the process and thus making it difficult to estimate the instantaneous correlation between individual processes. In addition, the treatment of censored effects is too difficult. Such effects produce problems in the autoregressive structure because the information on the exact length of the model is required for the continuation of the time series.

In this paper, we present the basic form of the conditional duration model (ACD) proposed by the authors of [2,7] since it is the most common type of autoregressive duration model and the most widely used in the recent econometric literature, and we discuss in more detail its theoretical properties and its estimation issues.

3.1. ARMA modeling for logarithmic variables

The best autoregressive model for variables with positive values $\left(x_{\{i\}} > 0\right)$ is to specify a logarithmic model for these variables ($\log x_{\{i\}}$), since these variables are not subject to non-negativity restrictions. The ARMA model for $log x_{\{i\}}$ is given by

$$log\left(x_{\{i\}}\right) = \omega + \sum_{j=1}^{p} \alpha_j log\, x_{\{i-j\}} + \sum_{j=1}^{q} \beta_j\, \tilde{\varepsilon}_{\{i-j\}} + \tilde{\varepsilon}_{\{i\}}, \quad i = 1, \ldots, n. \tag{3}$$

where $\varepsilon_{\{i\}}$ is white noise. If $x_{\{i\}}$ is a financial duration, the model belongs to the classes of AFT (Accelerated Failure Time)[1] models. Under the assumption of normality of $\varepsilon_{\{i\}}$, the parameters $\theta = (w, \alpha, \beta)'$ are estimated by the quasi-maximum likelihood method (QML). Thus, the conditional mean of $log x_{\{i\}}$ can be specified from (3) with a conditional variance $h_{\{i\}}$ following the GARCH model according to this relationship

$$\begin{cases} \varepsilon_{\{i\}} = \sqrt{h_{\{i\}} u_{\{i\}}}, \quad u_{\{i\}} \sim \aleph(0,1) \\ h_{\{i\}} = \sum_{j=1}^{p_k} \alpha_{h,j}\, \varepsilon_{\{i-j\}}^2 + \sum_{j=1}^{q_k} \beta_{h,j} h_{\{i-j\}} \end{cases} \tag{4}$$

Most research is not interested in modeling ($\log x_{\{i\}}$) but rather the variable $x_{\{i\}}$ developed by the ARMA model:

$$x_{\{i\}} = \omega + \sum_{j=1}^{p} \alpha_j x_{\{i-j\}} + \sum_{j=1}^{q} \beta_j \tilde{\varepsilon}_{\{i-j\}} + \tilde{\varepsilon}_{\{i\}} \tag{5}$$

with $w > 0$, $\alpha_{\{j\}} \geq 0$, $\beta_{\{j\}} \geq 0$. The estimator of the parameter vector $\theta = (w, \alpha, \beta)'$ is obtained by assuming

[1] More details.in [17].

that the $\varepsilon_{\{i\}}$ follow the exponential distribution determined by the QML

$$logL(X;\ \theta) = -\sum_{i=1}^{n} \varepsilon_{\{i\}} = -\sum_{i=1}^{n}\left(x_{\{i\}} - \omega + \sum_{j=1}^{p} \alpha_j x_{\{i-j\}} + \sum_{j=1}^{q} \beta_j \tilde{\varepsilon}_{\{i-j\}}\right) \quad (6)$$

in this case, a correct specification of the conditional mean ensures an efficient estimator of θ.

3.2. The ACD model

The authors [1,5,7] propose a MEM specification whose basic idea is the parameterization of the conditional mean

$$\psi_{\{i\}(\theta)} = E\left[x_{\{i\}}/F_{\{i-1\}}, \theta\right] \quad (7)$$

where θ is a column vector of p×1 parameters, $F_{\{i-1\}}$ is the past information related to the observation at $t_{\{i-1\}}$ while the durations $\varepsilon_{\{i\}} = \left(\frac{x_{\{i\}}}{\psi_{\{i\}}}\right)^2$ follow a positive definite iid process with $E[\varepsilon_{\{i\}}]=1$. The ACD model can be identified with the GARCH model and in the literature, there are different ACD models that differ according to the choices of the conditional mean $\psi_{\{i\}}$ or by the choice of the distribution of the perturbation term. The model is based on the linear parameterization of the conditional mean

$$\psi_{\{i\}} = \omega + \sum_{j=1}^{p} \alpha_j x_{\{i-j\}} + \sum_{j=1}^{q} \beta_j \psi_{\{i-j\}} \quad (8)$$

The conditional mean of the ACD model is given by definition by $E\left[x_{\{i\}}|F_{\{i-j\}}\right] = \psi_{\{i\}}$, while the mean and variance are given by

$$E\left(x_{\{i\}}\right) = E\left(\psi_{\{i\}}\right)E\left(\varepsilon_{\{i\}}\right) = \left(\frac{\omega}{1-\sum_{j=1}^{p} \alpha_j x_{\{i-j\}} + \sum_{j=1}^{q} \beta_j \psi_{\{i-j\}}}\right) \quad (9)$$

$$V\left[x_{\{i\}}|F_{\{i-1\}}\right] = \psi_{\{i\}}^2 \cdot V\left(\varepsilon_{\{i\}}\right) \quad (10)$$

For the case where p = q = 1, the variance is given by[3]

$$V\left(x_{\{i\}}\right) = E\left(x_{\{i\}}\right)^2 V\left(\varepsilon_{\{i\}}\right)\left[\left(\frac{1-\beta^2-2\,\alpha\,\beta}{1-(\alpha+\beta)^2-\alpha^2 V\left(\varepsilon_{\{i\}}\right)}\right)\right] \quad (11)$$

Yet, as can be noted, the stationarity conditions for the ACD model are similar to those for the GARCH model and are satisfied by $(\alpha+\beta)^2-\alpha^2 V(\varepsilon_{\{i\}})<1$. The corresponding results for the higher order ACD models are similar but more difficult to compute. It is easy to see that $V\left(x_{\{i\}}\right)>E\left(x_{\{i\}}^2\right)$. Thus, the ACD model involves excessive

[2] $\varepsilon_{\{i\}}$ and $\psi_{\{i\}}$ are two independent variables.

[3] We admit for this case, $\alpha=\alpha_1$ and $\beta=\beta_1$.

dispersion, i.e., the (unconditional) standard deviation exceeds the (unconditional) mean[4]. Similarly, the martingale difference $\eta_{\{i\}}=x_{\{i\}}-\psi_{\{i\}}$ may be introduced, the ACD (p, q) model can be written for the variable $x_{\{i\}}$ as an ARMA(max(p, q),q) model

$$x_{\{i\}} = \omega + \sum_{j=1}^{max(p,q)} \left(\alpha_{\{j\}} + \beta_{\{j\}}\right) x_{\{i-j\}} + \sum_{j=1}^{\{q\}} \beta \eta_{\{i-j\}} + \eta_{\{i\}}. \qquad (12)$$

3.3. The log-ACD model

This model was introduced by [19,20], it ensures the non-negativity of the durations without any restriction on the parameters and it is obtained by the multiplicative relation $x_{\{i\}} = \psi_{\{i\}} \varepsilon_{\{i\}}$

$$\begin{cases} log\psi_{\{i\}} = \omega + \alpha\ log\varepsilon_{\{i-1\}} + \beta\ log\psi_{\{i-1\}}, \\ log\psi_{\{i\}} = \omega + \alpha\ logx_{\{i-1\}} + (\beta - \alpha)\ log\psi_{\{i-1\}}, \end{cases} \qquad (13)$$

with $\varepsilon_{\{i\}}$ is iid and of expectation equal to one. We will refer to this model as the logarithmic ACD model of type 1 ($LACD_1$). In contrast to the linear ACD model, the LACD model involves a concavity relationship between $\varepsilon_{\{i-1\}}$ and $x_{\{i\}}$,, which is called the news impact curve. The difference in the impact of innovations with $\varepsilon_{\{i\}} < 1$ (bad news) on $x_{\{i\}}$ is larger than in the case of innovations with $\varepsilon_{\{i\}} > 1$ (good news). In the case of distributed innovation with a mean different from one, the process can be represented by

$$x_{\{i\}} = \psi_{\{i\}} \frac{\tilde{\varepsilon}_{\{i\}}}{\zeta} = \Phi_{\{i\}} \tilde{\varepsilon}_{\{i\}} \qquad (14)$$

$$log\Phi_{\{i\}} = \omega + \alpha log\varepsilon_{\{i-1\}} + \beta log\Phi_{\{i-1\}} \qquad (15)$$

with $\tilde{\omega} = \omega + (\beta - 1) log\zeta$ and $\varepsilon_{\{i\}} = \frac{\varepsilon_{\{i\}}}{\zeta}$ avec $E\left[\varepsilon_{\{i\}}\right] = \zeta \neq 1$. The Log-ACD model can similarly have an ARMA specification as developed in [21], who showed that from equation (15), the model will be an ARMA (R, R) process for the log $x_{\{i\}}$.

$$\log x_{\{i\}} = \tilde{\omega} + \sum_{j=1}^{R} \delta_j\ log\ x_{\{i-j\}} + \sum_{j=1}^{R} \theta_j\ \xi_{\{i-j\}} + \xi_{\{i\}} \qquad (16)$$

With

$$\xi_{\{i\}} = \left(log\varepsilon_{\{i\}} - E\left[log\varepsilon_{\{i\}}\right]\right) \sim iid\left(0, \sigma_{\{\xi\}}^2\right),$$

$$\tilde{\omega} = \omega + \sum_{j=1}^{R} \theta_j\ E\left[log\varepsilon_{\{i\}}\right] + E\left[log\varepsilon_{\{i\}}\right]\ and\ R = max(p,q).$$

Another alternative has been proposed by [19] for the Log-ACD model given by

[4] This property can be seen as the counterpart of the "flattening property" of the Gaussian GARCH model. General formulations of the lower and upper bounds of the d.e.f. of the duration process implied by an ACD (P,Q) model are given in [18].

$log\psi_{\{i\}} = \omega + \alpha\ \varepsilon_{\{i-1\}} + \beta\ log\psi_{\{i-1\}}$ and

$$log\psi_{\{i\}} = \omega + \alpha\ \frac{x_{\{i-1\}}}{\psi_{\{i-1\}}} + \beta\ log\psi_{\{i-1\}}. \qquad (17)$$

which involves a convex news impact curve that we call the Log-ACD Type II ($LACD_2$). These two models can be written as follows

$$log\psi_{\{i\}} = \omega + \sum_{j=1}^{p} \alpha_j\ g\left(\varepsilon_{\{i-1\}}\right) + \sum_{j=1}^{q} \beta_j\ log\psi_{\{i\}}\ , \qquad (18)$$

where $g\left(\varepsilon_{\{i\}}\right) = log(\varepsilon_{\{i\}})$ (type I), or $g\left(\varepsilon_{\{i\}}\right) = \varepsilon_{\{i\}}$ (type II). Bauwens [22] and Karanasos [23] determine the moments for the two logarithmic specifications of the ACD model. For the case $p = q = 1$, the moment of order r of $x_{\{i\}}$ is given by

$$E\left(x_{\{i\}}^{\{r\}}\right) = \mu_{\{r\}}\ exp\left(\frac{r\ \omega}{1-\beta}\right) \prod_{j=1}^{\infty} E\left(exp\left\{r\alpha\beta^{\{j-1\}} g\left(\varepsilon_{\{i\}}\right)\right\}\right), \qquad (19)$$

where r is a positive integer and the following conditions are satisfied,

$\mu_{\{r\}} = E\left(x_{\{i\}}^{\{r\}}\right) < \infty$, $|\beta|<1$, and $E\left(exp\left\{r\alpha\beta^{\{j-1\}} g\left(\varepsilon_{\{i\}}\right)\right\}\right) < \infty$.

As demonstrated by [21], the log-likelihood properties of the linear ACD model cannot be applied to the Log-ACD case, for the simple reason that the innovation $\varepsilon_{\{i-1\}}$ affects $\psi_{\{i\}}$ and $x_{\{i\}}$ in a non-linear manner and thus, destroys the validity of the log-likelihood score function in the case of a wrong formulation of the distribution. An alternative proposed by [21] to estimate the LACD model is to admit the log-normal distribution and assuming a normal distribution for the $\log x_{\{i\}}$ variable of mean $\psi_{\{i\}}$ and variance σ^2, the likelihood function is given by

$$lnL(X, \theta) = -\left(\frac{1}{2}\right) log2\pi - \left(\frac{1}{2}\right) log\sigma^2 - log\ x_{\{i\}} - \left(\frac{1}{2}\right)\left(\frac{\left(log\ x_{\{i\}} - log\psi_{\{i\}}\right)^2}{\sigma^2}\right) \qquad (20)$$

Another class of LACD model is the augmented LACD models that are presented in terms of a polynomial with random coefficients as analyzed by Carrasco [24] and is given by:

$$\vartheta\left(\psi_{\{i\}}\right) = A\left(\varepsilon_{\{i\}}\right)\vartheta\left(\psi_{\{i-1\}}\right) + B\left(\varepsilon_{\{i\}}\right) \qquad (21)$$

where $\vartheta(\cdot)$ is a continuous function on $[0, +\infty[$, A and B are two polynomials, and $\varepsilon_{\{i\}} = \frac{x_{\{i\}}}{\psi_{\{i\}}}$ is an iid innovation term with $E\left(\varepsilon_{\{i\}}\right) = 1$.

This class of model contains several extensions of the basic ACD model with additive and multiplicative stochastic components, i.e., the lagged innovation manifests itself at the mean by an additive or multiplicative pattern. For simplicity of the work, we will restrict ourselves only to lags of order 1 for P and Q.

3.4. The additive and multiplicative ACD model (AMACD)

An extension of the ACD model incorporating the two additive and multiplicative components of innovations is given by

$$\psi_{\{i\}} = \omega + \left(\alpha\psi_{\{i-1\}} + \nu\right)\varepsilon_{\{i-1\}} + \beta\psi_{\{i-1\}} \tag{22}$$

where ν is a parameter. This specification implies a news impact curve with a slope given by $\left(\alpha\psi_{\{i-1\}}+\nu\right)$. Here, the lag of the innovation term in the conditional mean manifests itself in an additive and multiplicative manner. In this sense, this AMACD model approximates the ACD model for ν=0.

3.5. The Box-Cox ACD (BACD) model

Hautsch [25] suggests an additive ACD model that is based on a power transformation of the parameters $\psi_{\{i\}}$ and $\varepsilon_{\{i-1\}}$:

$$\psi_{\{i\}}^{\{\delta_1\}} = \omega + \alpha\varepsilon_{\{i-1\}}^{\{\delta_2\}} + \beta\psi_{\{i-1\}}^{\{\delta_1\}} \tag{23}$$

where $\delta_1, \delta_2 > 0$. Since this chronicle exhibits long-run movements of the nonlinear (power) type, then it can be transformed by Box-Cox writing to recover the linearity of the model. This transformation is as follows for $\delta_1, \delta_2 > 0$:

$$\frac{\psi_{\{i\}}^{\{\delta_1\}}-1}{\delta_1} = \widetilde{\omega} + \tilde{\alpha}\,\frac{\left(\varepsilon_{\{i-1\}}^{\{\delta_2\}}-1\right)}{\delta_2} + \beta\,\frac{\psi_{\{i-1\}}^{\{\delta_1\}}-1}{\delta_1} \tag{24}$$

with $\widetilde{\omega} = ((\omega + \alpha + \beta - 1)/(\delta_1))$ and $\tilde{\alpha} = \frac{\alpha\delta_2}{\delta_1}$. This specification allows for concavity, convexity of functions, and linearity of new impact functions. For $\delta_1 = \delta_2 = 1$, we find the AMACD model, for $\delta_1 \to 0$, $\delta_2 \to 0$, we have the $LACD_1$ model and for $\delta_1 \to 0$, $\delta_2 = 1$ we find the $LACD_2$ model.

3.6. The Exponential ACD model (EXACD)

The authors in [26] introduce the Exponential ACD model with the same characteristics as the EGARCH model proposed by [27]. This model allows for linearity of the new impact curve which is formulated around $\varepsilon_{\{i-1\}} = 1$:

$$log\psi_{\{i\}} = \omega + \alpha\varepsilon_{\{i-1\}} + c\left|\varepsilon_{\{i-1\}} - 1\right| + \beta log\psi_{\{i-1\}} \tag{25}$$

With c a negative rotation parameter of the new impacts curve (nic)

If the durations are too small than the conditional mean $\left(\varepsilon_{\{i-1\}} < 1\right)$, the new impacts curve has a slope equal to $\alpha - c$ and a trend equal to $\omega + c$, whereas for durations greater than the conditional mean $\left(\varepsilon_{\{i-1\}} > 1\right)$, the slope and trend of the curve are, respectively, $\alpha + c$ and $\omega + c$.

3.7. The SNIACD (Spline News Impact ACD) model

This involves modeling the new responses in terms of a piecewise linear function. In the spirit of [28], the new impact curve can be modeled as a linear spline function with nodes for each breakpoint $\varepsilon_{\{i-1\}}$. In particular, the interval of $\varepsilon_{\{i-1\}}$ is divided into M intervals, where $M^{-}(M^{+})$ denotes the number of intervals in $\varepsilon_{\{i-1\}} < 1\ (\varepsilon_{\{i-1\}} > 1)$ with $M = M^{-} + M^{+}$. Noting the breakpoints by $\{\varepsilon_{\{M^{-}\}}, \dots, \varepsilon_{\{-1\}}, \varepsilon_{\{1\}}, \dots, \varepsilon_{\{M^{+}\}}\}$, the SNIACD model is given by

$$\psi_{\{i\}} = \omega + \sum_{\{m=0\}}^{\{M^{+}\}} \alpha_{\{m\}}^{+}\, ¶_{\{\{\varepsilon_{\{i-1\}} \geq \varepsilon_{\{m\}}\}\}}\left(\varepsilon_{\{i-1\}} - \bar{\varepsilon}_{\{m\}}\right) + \sum_{\{m=0\}}^{\{M^{-}\}} \alpha_{\{m\}}^{-}\, ¶_{\{\{\varepsilon_{\{i-1\}} < \varepsilon_{\{m\}}\}\}}\left(\varepsilon_{\{i-1\}} - \bar{\varepsilon}_{\{m\}}\right) + \beta\, \psi_{\{i-1\}} \quad (26)$$

where $\alpha_{\{m\}}^{+}$ and $\alpha_{\{m\}}^{-}$ are the coefficients associated with the piecewise linear spline function. One can specify the logarithmic transformation to the model and thus, one requires more non-negativity restrictions. The authors [28], show that for a small increase in M as a function of sample size should asymptotically yield an efficient estimator of the new impact curve.

Thus, and after presenting all these models and returning to equation (22), we will be able to classify all these duration models using a good parameterization of the functions $A(\cdot), \vartheta(\cdot)$ and $B(\cdot)$ (see **Table 4**).

Table 4: Differents ACD processes

	$\vartheta(\psi_{\{i\}})$	$A(\varepsilon_{\{i\}})$	$B(\varepsilon_{\{i\}})$
Linear and logarithmic models			
ACD	$\psi_{\{i\}}$	$\alpha\varepsilon_{\{i-1\}} + \beta$	ω
LACD1	$\log \psi_{\{i\}}$	β	$\omega + \alpha \log \varepsilon_{\{i-1\}}$
LACD2	$\log \psi_{\{i\}}$	β	$\omega + \alpha\ \varepsilon_{\{i-1\}}$
Nonlinear models			
EXACD	$\log \psi_{\{i\}}$	β	$\omega + \alpha\ \varepsilon_{\{i-1\}} + c\lvert\varepsilon_{\{i-1\}} - 1\rvert$
ABACD	$\psi_{\{i\}}^{\{\delta_1\}}$	β	$\omega + \alpha\ \left(\lvert\varepsilon_{\{i-1\}} - b\rvert + c(\varepsilon_{\{i-1\}} - b)\right)^{\{\delta_2\}}$
Augmented ACD Models			
AMACD	$\psi_{\{i\}}$	$\alpha\varepsilon_{\{i-1\}} + \beta$	$\omega + \nu\ \varepsilon_{\{i-1\}}$
SNIACD models			
SNIACD	$\psi_{\{i\}}\ or\ \log \psi_{\{i\}}$	β	$\omega + \sum_{\{m=0\}}^{\{M^{+}\}} \alpha_{\{m\}}^{+}\, ¶_{\{\{\varepsilon_{\{i-1\}} \geq \varepsilon_{\{m\}}\}\}}\left(\varepsilon_{\{i-1\}} - \bar{\varepsilon}_{\{m\}}\right) + \sum_{\{m=0\}}^{\{M^{-}\}} \alpha_{\{m\}}^{-}\, ¶_{\{\{\varepsilon_{\{i-1\}} < \varepsilon_{\{m\}}\}\}}\left(\varepsilon_{\{i-1\}} - \bar{\varepsilon}_{\{m\}}\right)$

Source: [17]

4. *Estimation* and interpretation of results

Before presenting the estimation results of the different models, and since we are interested in modeling the time between two trades, we started by calculating the duration in seconds between two trades $x_{\{i\}} = t_{\{i\}} - t_{\{i-1\}}$ for the two securities Adwya and Uib. When data are recorded in seconds, such precision allows us to aggregate trades made at the same second and at the same time eliminate zero durations. As shown in the **figure 4**, the durations between transactions present a clustering phenomenon, such that we observe strong moments of exchange followed by strong moments of exchange and weak moments of exchange followed by weak moments of exchange. However, the transaction durations have a clear daily structure because the level of activity varies during the day, in particular, this activity is very strong at the beginning and end of the day and this then allows a variation of the average over time. Therefore, we observe in this chronicle, a daily seasonal variation which is modeled in the literature by a seasonal factor $\emptyset_{\{j\}}\left(\tau_{\{t_{\{i\}}\}}\right)$ (Diurnally pattern or also deterministic term).

To obtain stationarity of the series, this seasonal factor will be modeled by a cubic spline function[5] at the time of the realization of each transaction, that is, we have calculated at each moment when a transaction is carried out a seasonal coefficient that will be adopted for all days of the week (See **Table.2**). The seasonal coefficient is obtained by splitting the spline function into four nodes of one hour each and counting from midnight ($10\ h = 36000\ seconds$). The series will be stationary after removing the seasonal coefficient from the raw series and the resulting series is called the seasonally adjusted $\tilde{x}_{\{t_{\{i\}}\}} = \frac{x_{t_{\{i\}}}}{\emptyset_{\{j\}}\tau_{t_{\{i\}}}}$ (Diurnally adjusted durations).

4.1. Empirical illustrations of the different models

In this paragraph, we will estimate a panoply of the different ACD specifications for the two stocks Adwya and Uib covering the period from 01/01/2014 to 31/06/2014. For each financial term, six ACD specifications are selected with exponential and Weibull distributions for the error term[6].

We suggested these two specifications of distributions for the error term since they are the most used for this type of data. The estimation of the different models is performed by the quasi-maximum likelihood method (QML) for the error with exponential distribution and by maximum likelihood for the weibull distribution, while, the choice of the delay is defined by the Bayes information criterion (BIC). **Tables 5** and **6** gives the estimation results of the different models knowing that the SNIACD model in [17] was changed slightly according to the Belfrage [28] model.[7]

By analyzing the estimation results of the different models, some conclusions can be drawn. All the coefficients of the different models are significant at the 5% level except for the gamma parameter of the AMACD model of the Uib stock.

With no negativity restriction on the parameters of the different regressions the coefficients associated with the

[5] $\emptyset\left(t_{\{i\}}\right) = \xi^3_{\{k,3\}}\left(t_{\{i\}} - t_{\{k\}}\right) + \xi^2_{\{k,2\}}\left(t_{\{i\}} - t_{\{k\}}\right) + \xi_{\{k,1\}}\left(t_{\{i\}} - t_{\{k\}}\right) + \xi_{\{k,0\}}$ with $t_{\{k\}}$ is the kth node.

[6] See the appendix for the two distributions.

[7] See the package under R in [28].

innovations term are too small and for some models where the coefficient α_2 is negative, this is overcompensated by the positivity of the parameter α_1 ($0 \leq \alpha_1 + \alpha_2 \leq 0{,}5$), while the persistence parameters are close to unity. Nevertheless, from the estimates, we find no violation of the conditional mean non-negativity restriction.

Comparing the goodness of fit of the models through the BIC value, we find that the two logarithmic models have the better specification than the others for the Adwya stock. On the other hand, for the Uib stock, the best specification is given to the $LACD_1$ model, nevertheless, for all these specifications, we found a strong increase in the log likelihood function and thus, this result is obvious since the non-linear modeling of the different models is only an extension of the linear models such as the ACD and $LACD_{1,2}$ models which is an important and crucial specification to take into account the non-linear effects of the new impacts. Thus, for Adwya stock the WABACD and WSNIACD models are the best fits to the data, however, for Uib stock the WABACD model is the best specification.

Table 5: Estimation of the different ACD models of Adwya stock with breakpoints for the SNIACD model **0.5, 1.5**

	ACD	**ACD**	**LACD1**	**LACD2**	**AMACD**	**EXACD**	**BACD**	**WABACD**	**WLSNIACD**
ω	0.00806 (0.001)	0.0033 (0.00052)	0.1231 (0.0080)	−0.0735 (0.005)	−0.00098 (0.00215	−0.05183 (0.00507	−0.0897 (0.00935	−0.0490 (0.00374)	−0.308 (0.0139)
α_1	0.13 (0.01)	0.2928 (0.023)	0.0948 (0.00606	0.0677 (0.00430	0.132992 (0.01299	0.9782 (0.00269	0.1410 (0.01217	0.1067 (0.00380)	
α_2		−0.2215 (0.023)						−0.0281 (0.00461)	
β_1	0.87309 (0.00832	0.9299 (0.00607)	0.9875 (0.00231	0.9865 (0.00215	0.424491 (0.078)	0.1813 (0.01242	0.9832 (0.0032)	0.6137 (0.00903)	0.970 (0.006)
β_2					0.429787 (0.07292			0.3676 (0.00888)	
c_0									0.714 (0.03771)
c_1						−0.1221 (0.01253		0.6315 (0.008)	−0.517 (0.06368)
c_2								0.8581 (0.00886)	−0.138 (0.04198)
ν					0.018173 (0.00368				
δ_1							0.5160 (0.0397)	0.236 (0.007)	
δ_2							0.4086 (0.028)	0.373 (0.007)	
γ								0.6262 (0.00756)	0.616 (0.00744)
b								0.0666 (0.00288)	
LL	-1929.8	-1865.17	-2114	-1999.1	-1892.3	-1930.4	-1884	-821.986	-846.239
BIC	3884.14	3763.08	4252.49	4022.81	3825.53	3893.69	3808.86	1734.015	1741.592
AIC	3865.58	3738.34	4233.94	4004.25	3794.6	3868.8	3777.93	1665.973	1704.479

Concerning the two estimated parameters δ_1 and δ_2, their sums are less than one for the Adwya stock while, for the Uib stock we have a sum greater than one for both BACD and WABACD models which is in conflict with the linearity of the ACD models especially the $LACD_{1,2}$ models. Note also, that the concavity of the curve of new impacts is well verified, the coefficient c_1 of the two headings of the EXACD model is negative, this means that negative shocks ($\varepsilon_{\{i-1\}} < 1$) on the conditional mean act more violently than positive shocks ($\varepsilon_{\{i-1\}} > 1$), in other words, bad information only increases the waiting time of the trade realization time and vice versa.

Thus, for the WABACD and ABACD models, we have evidence for both stocks of a concavity of the news impact curve since the value of δ_2<1. Whereas, for the last SNIACD model, it is estimated using breakpoints for both headings {0.5,1.5} which allow more flexibility for strong and weak innovations. All coefficients are significant at 5% for different thresholds. As can be seen from the second lag of the coefficient c_2, flexibility for new impacts is achieved.

Table 6: Estimation of the different ACD models of Uib stock with breakpoints for the SNIACD model **0.5, 1.5**

	ACD	**LACD1**	**LACD2**	**AMACD**	**BACD**	**EXACD**	**WABACD**	**ABACD**	**SNIACD**
ω	0.127 (0.0175)	0.232 (0.0164)	−0.0654 (0.00909)	0.00871 (0.00496)	−0.341 (0.0897)	−0.0668 (0.01453)	−0.1306 (0.00511)	0.07277 (0.00351)	−0.308 (0.01390)
α_1	0.254 (0.0265)	0.207 (0.0115)	0.0617 (0.00821)	0.08983 (0.01995)	0.664 (0.0881)	0.8140 (0.025)	0.4244 (0.0062)	0.17605 (0.00288)	
α_2							−0.0813 (0.00536)		
β_1	0.638 (0.0354)	0.763 (0.0219)	0.9775 (0.00670)	0.51184 (0.07856)	0.825 (0.0189)	0.322 (0.0272)	0.9109 (0.00673)	0.79349 (0.00357)	0.8415 (0.00575)
β_2				0.34779 (0.07276)				−0.0686 (0.00576)	
c_0									0.5094 (0.02658)
c_1						−0.2474 (0.02715)	0.1452 (0.00794)	−0.17938 (0.00434)	−0.2025 (0.066)
c_2							0.3139 (0.0106)		−0.2627 (0.0608)
ν				0.04358 (0.01342)					
δ_1					0.863 (0.0487)		0.7739 (0.0108)	0.17836 (0.004)	
δ_2					0.290 (0.0617)		0.4332 (0.007)	0.26783 (0.00385)	
b							0.0231 (0.00261)	0.00436 (0.00122)	
γ							0.6219 (0.00913)		
LL	-1933.03	-1930.8	-1952.97	-1924.56	-1844.48	-1909.475	-1173.742	-1892.368	-1859.1
BIC	3889.301	3884.83	3929.19	3887.87	3727.686	3849.935	2432.694	3838.96	3756.9273
AIC	3872.062	3867.6	3911.957	3859.138	3698.955	3826.95	2369.484	3798.736	3728.196

4.2. Estimation results for Aggregate Data

Transaction data is often used in an aggregated fashion. Although this natural process results in a loss of information, there are three reasons for using such a process. First, temporal aggregation allows the construction of relevant economic variables of interest. Second, temporal aggregation reduces the impact of the market microstructure effect whenever the latter is losing interest and causing noise. Third, temporal aggregation reduces the amount of useful data whenever the study period is long or when a large cross-section is involved. In general, we distinguish between two major types of sampling and aggregation:

1. Event aggregation or aggregation of the process according to the specific events of the exchange.

2. Temporal aggregation or aggregation of the specific process in calendar time.

In this section, we consider temporal aggregation for the 5- and 30-minute time intervals that will be applied to the ACD, EXACD and SNIACD models. The choice of these two different time intervals was based on the fact that it was found that between an aggregation of 30 seconds and 2 minutes the data did not change and this is due to the long waiting time between the transactions. **Tables 7** and **8** provide the results of estimates of the aggregate seasonally adjusted durations of both titles. The estimation of the three models by quasi-maximum likelihood with exponential distribution error shows a persistence of the parameters accompanied by very clear evidence of the asymmetry of the shocks as indicated by the EXACD model.

Table 7: Aggregate estimation of the different ACD models of Adwya stock with breakpoints for the SNIACD 0.5 and 1.5 model.

	5min		30min	
Coefficients	Estimates	P values	Estimates	P values
ACD				
w	0.007	0.003	0.0085	0.134
α	0.06	0.000	0.0391	0.000
β	0.9321	0.000	0.9581	0.000
BIC	2792.16		2110.33	
LL	-1385.5		-1045.52	
EXACD				
w	-0.03	0.001	-0.008	0.55
α	0.085	7.03e-13	0.07	8.6e-06
c	-0.053	0.0002	-0.05	0.031
β	0.99	0.000	0.993	0.000
BIC	3644.45		2126.93	
LL	-1807.727		-1050.608	
SNIACD				
ω	-0.014	0.000	-0.0290	0.000
c_0	-0.198	0.000	-0.0828	0.000
c_1	0.556	0.000	0.4235	0.000
c_2	-0.355	0.000	-0.4248	0.000
β	0.982	0.000	0.9935	0.000
BIC	3722.615		2104.45	
LL	-1843.15		-1036.15	
N.Obs	1425		620	

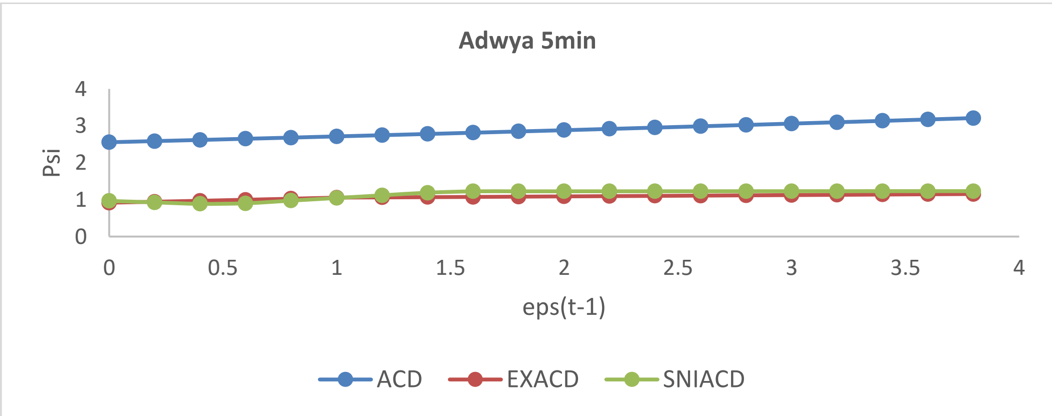
Table 8: Aggregate estimation of the different ACD models of Uib stock with breakpoints for the SNIACD 0.5 and 1.5 model

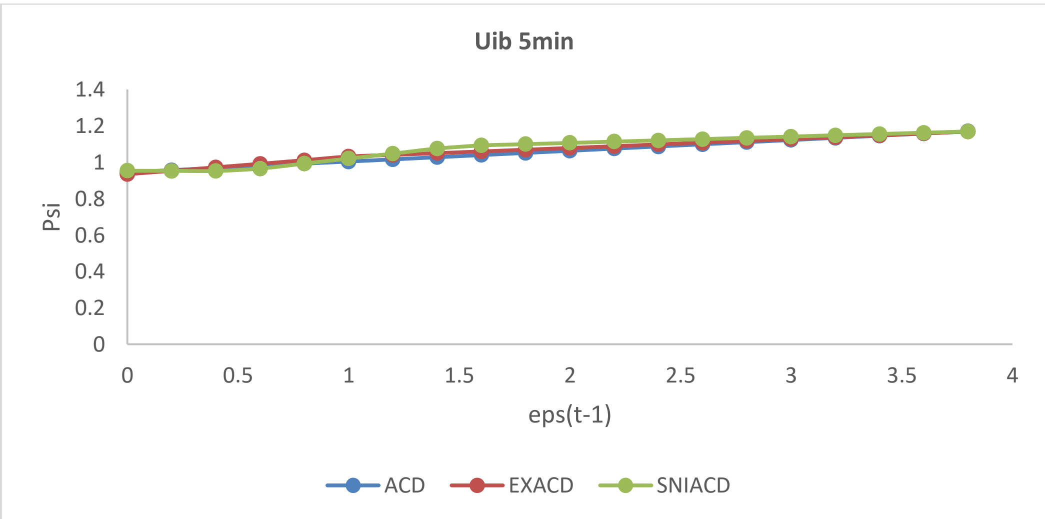
	5min		30min	
Coefficients	Estimates	P values	Estimates	P values
ACD				
w	0.0134	0.005	0.0156	0.032
α	0.0592	0.000	0.0693	0.000
β	0.9321	0.000	0.9229	0.000
BIC	2792.16		1501.3	
LL	-1385.5		-741.2	
EXACD				
w	-0.040	2.73e-05	-0.043	0.007
α	0.0716	2.76e-06	0.08	0.002
c	-0.027	0.12	-0.03	0.344
β	0.99	0.000	0.99	0.000
BIC	2800.93		1509	
LL	-1386.354		-741.9	
SNIACD				
ω	0.029-	0.000	0.110-	0.000
c_0	0.003-	0.865	0.328	0.000
c_1	0.140	0.000	0.396-	0.000
c_2	0.102-	0.000	0.157	0.009
β	0.983	0.000	0.996	0.000
BIC	2807.064		1503.93	
LL	-1385.9		-736.21	
N.Obs	1159		546	

The estimate of the coefficient c is significant and of negative sign which reveals the concavity of the shock function which means that the marginal effect of long durations on the conditional mean measured by $(\omega + c)$ is weaker than the marginal effect of shorter durations measured by $(\omega - c)$, i.e., on the market, as the time between trades becomes longer and longer, investors will be less and less sensitive to the time passages. From a goodness-of-fit point of view, there is not much difference between the three models, which is also confirmed by the log-likelihood. These effects are further confirmed by **figure 4** which describes the impact of new shocks to these three models.

We set $\psi_{\{i-1\}}$ to its unconditional mean (marginal mean) and let $\varepsilon_{\{i-1\}}$ vary in order to assess its impact on the conditional mean $\psi_{\{i\}}$.

As can be seen, the linearity of the ACD model is observed for both stocks and for the two-time intervals considered. For the EXACD model, we observe clear evidence for the asymmetry of the impact curve which is concave ($\varepsilon_{\{i-1\}} < 1$) and, when $\varepsilon_{\{i-1\}} \geq 1$, the new impact curve is kinked upwards (with a negative rotation coefficient c. ($c < 0$)). The EXACD model predicts a shorter duration than the linear ACD model for both short and long shocks. We observe that the non- linearity of the curve is observed especially for small shocks we even observe a news response function that implies a downward shape for low values of $\varepsilon_{\{i-1\}}$ and this was confirmed in [2]. The shape of the impact curve of the SNIACD model also confirms these results, since for shocks $\varepsilon_{\{i-1\}} < 1.5$ for both stocks and as shown in **Figure 4**, the magnitude of the response to weak shocks is greater than for shocks $\varepsilon_{\{i-1\}} \geq 1.5$ since after this level, the curve becomes constant for the Adwya stock and linear in shape for the Uib stock and this for the two different time intervals. This result shows that the reaction of investors to shocks of small durations is faster, this is explained by the fact that when several informed operate in the market and if one of these participants refrains from taking immediate advantage of a trade and thus to wait for new market liquidity in the future, someone else will take his place to make even a limited profit since competition prevents the waiting strategy.





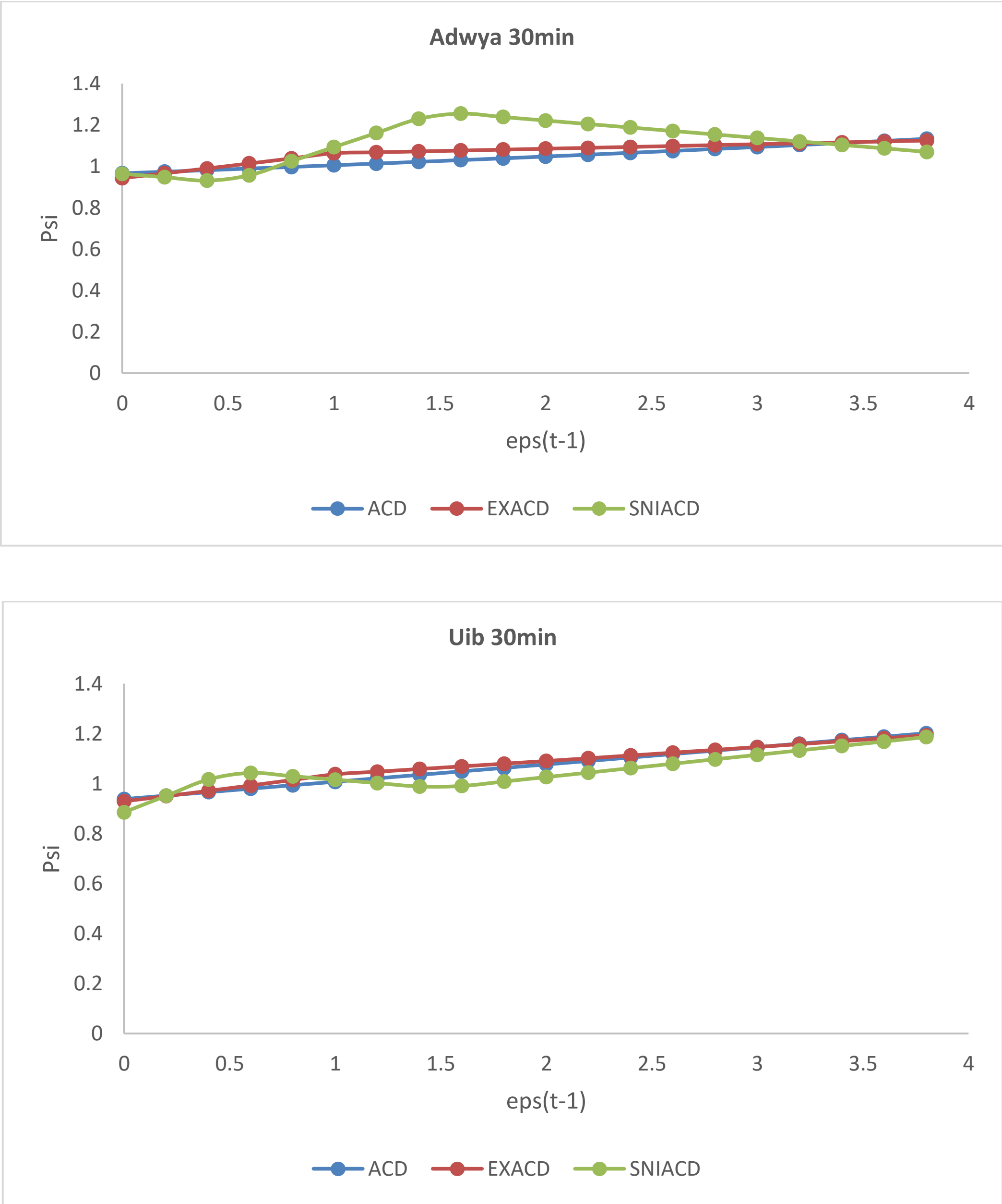


Figure 4: ACD Models for aggregation time data

5. Conclusion

In this paper, we model high frequency data for which the time of occurrence of each event is a random variable. These models are based on the assumptions of the distribution of the durations studied and in which the conditional expectation has an autoregressive form. Our study focused on the estimation of MEM multiplicative error models. We have estimated a set of models whose main specification is to determine the shape of the new impacts curve. This specification admits a concave form for the non-linear models such as the EXACD model and the SNIACD model while the linearity for the ACD model has been preserved for both titles.

The estimation of the different models is performed by quasi-maximum likelihood with an exponential or Weibull distribution of the error term. The results are rather satisfactory, all the coefficients are significant at the 5% level except for the gamma parameter of the AMACD model of the Uib stock. Similarly, we have a negative value of the coefficient c_1 of the EXACD model and thus a curve of new impacts which means that small shocks act with a very large magnitude on the conditional mean. For both coefficients $\delta_{\{i\}} < 1$ $(i = 1,2)$ of both BACD and ABACD models the curve is concave. The coefficients of the SNIACD model are significant allowing more flexibility in the case of small and large innovations.

In the second part of the chapter, we aggregated the 5- and 30-minute data applied to the ACD, EXACD and SNIACD models. The estimation of the last two models shows a persistence of the parameters, i.e., a persistence of the long and short durations between exchanges, as well as an asymmetry of the shocks explained by the concavity of the curve and the negative coefficient c. This evidence was well observed at the level of the graphs of the two titles for these different temporal aggregations. Thus, the new impact curve appears to be a generous way to capture the nonlinear dynamics of the ACD to MEM models. These effects are confirmed by the EXACD and SNIACD models, which are in our view the best models capable of capturing the non-linear dynamics of MEM models for fixed time intervals.

Any work done has its limitations and our limitations in this work are that we have neglected the data concerning the opening and closing fix, since we have processed the exchanges continuously which results in a loss of information. In addition, in future research on the various ACD models, the intraday seasonal adjustment technique appears to have a significant, but not well understood, effect on any model used. The elimination of zero durations from observations with the same time stamp seems to us to be a biased technique. We chose to work on overly active stocks, whereas empirical studies have shown that the impact is greater on stocks that are not very active in active and quiet periods had an over-reaction effect.

APPENDIX

Distributions for the error terms

Weibull distribution: ($\varepsilon \sim W((1/\lambda), \gamma)$),

$f(\varepsilon) = \theta\gamma\varepsilon^{\{\gamma-1\}}e^{\{-\theta\varepsilon^{\{\gamma\}}\}}$ for $\theta, \gamma > 0$, with $\theta = \left(\frac{1}{\lambda}\right)^{\{\gamma\}}$

Exponential distribution: ($\varepsilon \sim E(1/\lambda)$),

$f(\varepsilon) = (1/\lambda)exp(-(\varepsilon/\lambda))$ for $\lambda > 0$

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