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Transfer of Heat of a Jeffrey Fluid over a Linearly Stretching Sheet with Chemical Reaction: Numerical Study

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Abstract

This paper numerically explores the effect of heat transfer on Jeffrey fluid flow over a horizontal stretching sheet in the absence of magnetic field and under chemical reaction. The governing coupled nonlinear momentum, thermal, concentration boundary layer equations are rendered into a system of coupled nonlinear ordinary differential equations through similarity transformation with suitable boundary conditions. The obtained fourth order and second order differential equations are reduced to first order ordinary differential equations using shooting method then it is numerically solved using bvp4c in MATLAB. This present investigation is of great interest relevant to colling of metallic plates, polishing of artificial heart valves and separation processes in chemical industries.

Keywords: Jeffery fluid; Heat transfer; Chemical Reaction.

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1. Introduction

Heat transfer over a stretching sheet is important because the rate of cooling has great effect on the quality of the product. Boundary layer flow has extensive application in industry, Engineering, aerospace manufacturing, and medical Industries, numerous researchers and scientists achieved results in the fluid flow and heat transfer , M. R. Eid, and his colleagues [1] has given intense study on “Effects of NP Shapes on Non-Newtonian Bio-Nanofluid Flow in Suction/Blowing Process with Convective Condition: Sisko Model,” J. Nonequilibrium Thermodyn, M. J. Kotresh, G. K. Ramesh, V. K. R. Shashikala, and B. C. Prasannakumara, and his colleagues [2] has studied the “Assessment of Arrhenius activation energy in stretched flow of nanofluid over a rotating disc,” M. Sheikholeslami, S. A. Shehzad, Z. Li, andA. Shafee, and his colleagues [3] gave a “Numerical modeling for alumina nanofluid magnetohydrodynamic convective heat transfer in a permeable medium using Darcy law,” M. R. Eid and A. F. Al-Hossainy, and his colleagues [4] has given a research on “Synthesis, DFT calculations, and heat transfer performance large-surface TiO2: ethylene glycol nanofluid and coolant applications,’’ H.Blasius,Grenzschichten , and his colleagues [5] intense study in Flussigkeiten mit Kleiner Reibung,z.Angew, Fang, X.; Xuan, Y.; Li, Q, and his colleagues [6] gave an Experimental investigation on enhanced mass transfer in nanofluids, Veilleux, J.; Coulombe, S. , and his colleagues [7] gave A dispersion model of enhanced mass diffusion in nanofluids,Hayat, T., Mustafa, M., , and his colleagues [8] have studied the Influence of thermal radiation on the unsteady mixed convection flow of a Jeffrey fluid over a stretching sheet, Nadeem, S., Tahir, B., Labropulu, F., Akbar, N.S. and his colleagues [9] given a study on Unsteady oscillatory stagnation point flow of a Jeffrey fluid, Hayat, T., Shehzad, S.A., Qasim, M., Obaidat, S., , and his colleagues [10] pioneered the Radiative flow of Jeffery fluid in a porous medium with power law heat flux and heat source, M. Qasim, I. Khan, S. Sharidan, , and his colleagues [11] examined the Heat transfer in a micropolar fluid over a stretching sheet with Newtonian heating,S. Nadeem, R. Mehmood, Noreen. Sher Akbar, and his colleagues [12] examined the Non-orthogonal stagnation point flow of a nano non-Newtonian fluid towards a stretching surface with heat transfer, S. Nadeem, Noreen. Sher Akbar , and his colleagues [13] studied the Peristaltic flow of a Jeffrey fluid with variable viscosity in an asymmetric channel , M. Khan, F. Iftikhar, A. Anjum , and his colleagues [14] added Some unsteady flows of a Jeffrey fluid between two side walls over a plane wall ,T. Hayat, S. Asad, M. Qasim, A. Hendi , and his colleagues [15] gave Boundary layer flow of a Jeffrey fluid with convective boundary conditions,T. Hayat, S.A. Shehzad, M. Qasim, S.Obaid , and his colleagues [16] gave a study on Thermal radiation effects on the mixed convection stagnation-point flow in a Jeffery fluid, S. Srinivas and M. Kothandapani, and his colleagues [17] studied heat and mass transfer of fluid,M. Massoudi and I. Christie, and his colleagues [18,22] a study on Nonlinear steady flow, M Y Malik,I Zehra,S.Nadeem, and his colleagues Reference [23] gave a Numerical treatment of Jeffrey fluid with pressure dependent viscosity,C.S.K Raju ,M.J.Babu,N.Sandeep, and his colleagues [24] has examined the Chemically reacting radiative MHD Jeffrey nanofluid flow over a cone in porous medium, P.V Satya Narayana, D Harish Babu, and his colleagues [25] pioneered the Numerical study of MHD heat and mass transfer of a Jeffrey fluid over a stretching sheet with chemical reaction and thermal radiation several investigations have been carried on Jeffery fluid few of them are hereby cited. In view of all the mentioned above research the main objective of the present article is to explore the eat and mass transfer on the Jeffrey fluid in the absence of magnetic field over a linearly stretching sheet. Effect of non -dimensional governing parameters such as Prandtl number, the ratio of relaxation to retardation times. Here we also describe the numerical method, we present result and discuss. Finally, we summarize our result and present our conclusion.

1.1. Mathematical formulations

The essential equations for Jeffrey fluid can be written as

$$\tau = -pl + \mathrm{E} \tag{1}$$

$$\mathrm{E} = \frac{\mu}{1+\lambda_1}\left[R_1 + \lambda_2\left(\frac{\partial R_1}{\partial t} + V.\Delta\right)R_1\right] \tag{2}$$

Where E is the extra stress tensor, τ is the Cauchy stress tensor, $\lambda_1$and $\lambda_2$are the material parameters of Jeffrey fluid and R_1 is the Rilin -Ericksen tensor defined by

$$R_1 = (\nabla V) + (\nabla V)'$$

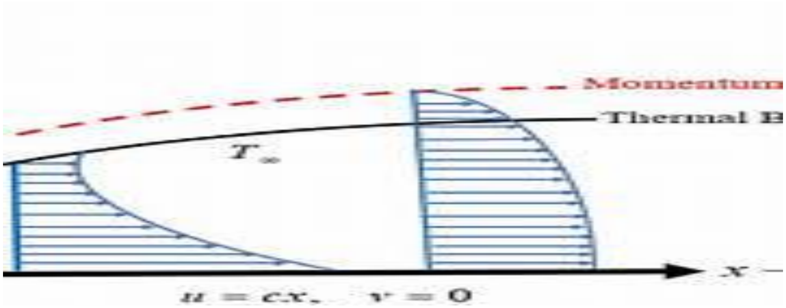

Figure 1: The physical model of the flow problem and coordinate system

A steady two-dimensional incompressible, electrically conducting Jeffrey fluid over a linear stretching sheet in the presence of chemical reaction, thermal radiation and heat source flow is generated,due to linear stretching of the sheet, caused by simultaneous application of two equal and opposite forces along the *x*-axis and *y*-axis is taken normal to it. Theorigin is fixed as shown in Figure 1.The temperature and the species concentration have power index *m* variations with the distance from the origin. At *t* = 0, the sheet is impetuously stretched with the variable velocity $U_w(x)$.

Under these assumptions the governing equation of continuity and momentum take the following form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{v}{1+\lambda_1}\left[\frac{\partial^2 u}{\partial y^2} + \lambda_2\left(u\frac{\partial^3 u}{\partial x\partial y^2} + v\frac{\partial^3 u}{\partial y^3} - \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right)\right] \tag{4}$$

Where u, v are the velocity components in the x and y direction, respectively,v is the kinematic viscosity, λ_1 is the ratio of relaxation and retardation time,λ_2 is the relaxation time.

The equation of heat transfer and thermal radiation is given as

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_r}{\partial y}-Q(T-T_\infty) \quad (5)$$

Where c_p is the specific heat and k is the thermal conductivity, T is the temperature of the fluid.T_∞ is the constant temperature of the fluid far away from the sheet

By using Rosseland diffusion approximation, the radiative heat flux q_r is given by

$$q_r=-\frac{4\sigma^*}{3K_s}\frac{\partial T^4}{\partial y} \quad (6)$$

Where K_s and σ^* are the Rosse land mean absorption coefficient and the Stefen-Boltzmann constant, resp. the temperature within the fluid flow is considered sufficiently small such that T^4can be expressed as linear function of temperature.

$$T^4\approx 4T_\infty{}^3T-3T_\infty{}^4 \quad (7)$$

On solving (6) (7) and (5) we get

$$\frac{\partial q_r}{\partial y}=-\frac{16\sigma^* T_\infty{}^3}{3K_s}\frac{\partial^2 T}{\partial y^2} \quad (8)$$

We introduce a dimensionless temperature variable $\theta(\xi)$ of the form

$$\theta(\xi)=\frac{T-T_\infty}{T_w-T_\infty} \quad (9)$$

1.2. Boundary Conditions

The following boundary conditions on velocity, temperature and concentration are appropriate in order to employ the effect of stretching of the boundary surface causing flow in x-direction as

$u=U_w(x)=cx, v=0\ at\ y=0$

$u\rightarrow 0, u'\rightarrow 0\ as\ y\rightarrow\infty$

$T=T_w=T_\infty+A_1\left(\frac{x}{l}\right)^m$ at y=0

$$T\rightarrow T_\infty \text{ as } y\rightarrow\infty \quad (10)$$

Where$A_{1,}$ A_2 are constants, l is the characteristic length, m is the surface temperature parameter, T_w is the stretching sheet temperature, C_w *and* C_∞ are the concentration at the wall and far away from the wall, resp.

The following similarity transformations are introduced to solve equation (4) (5) and (9)

$$u = cxf'(\xi), v = -\sqrt{cv}f(\xi) \; where \; \xi = \sqrt{\frac{c}{v}}y \qquad (11)$$

Where ξ is the similarity variable and $f(\xi)$ is the dimensionless stream function

Substituting eq (13) in eq (4) (5) and (10) we obtain second and fourth order ordinary differential equations as follows

$$f''' + (1+\lambda_1)\left(ff'' - f'^2\right) + \beta\left(f''^2 - ff^{iv}\right) = 0 \qquad (12)$$

$$\left(1 + \frac{4R}{3}\right)\theta'' + \Pr(f\theta' - mf'\theta + \gamma\theta) = 0 \qquad (13)$$

With boundary conditions (10) takes the form:

$$f(\xi) = s, f'(\xi) = 1 \; at \; \xi = 0; f'(\xi) = 0, f''(\xi) = 0 \; as \; \xi \to \infty$$

$$\theta(\xi) = 1 \; at \; \xi = 0 \; ; \; \theta(\xi) = 0 \; as \; \xi \to \infty \qquad (14)$$

Where $\beta = \lambda_2 c$ is the Deborah number, $R = \frac{4\sigma^* {T_\infty}^3}{K_s}$ the radiation parameter,

$Pr = \frac{\rho c_p}{k}$ the Prandtl number, $\gamma = \frac{Qv}{\rho c_p}$ is a heat source parameter

The system of non -linear ordinary differential equations (14) (15) (16) with the boundary conditions (17) are converted to ordinary differential equations using shooting method and using MATLAB bvp4c the numerical solution is obtained; thus, the fourth order and second order equations are reduced to system of simultaneous equations of order one.

$$f = y(1), f' = y(2), f'' = y(3), f''' = y(4)$$

$$\theta = y(5), \theta' = y(6) \qquad (15)$$

Substituting these in (11)(12)(13) and (14) we have

$$y(4) + (1+\lambda)(y(1)y(3) - y(2)^2) + \beta(y(3)^2 - y(1)f^{iv}) = 0 \qquad (16)$$

$$\theta''\left(1 + \frac{4}{3R}\right) - Pr(y(1)y(6) - my(2)y(5) + \gamma y(5)) = 0 \qquad (17)$$

Boundary Conditions

$y_0(1) = 1, y_0(2) = 1\ ;\ y_\infty(2) = 0, y_\infty(3) = 0; y_0(5) = 1, y_\infty(5) = 0; y_0(7) = 1, y_\infty(7) = 0;$

Equations (19)(20)(21) are reduced to eight simultaneous equations of first order as follows

$y'(1) = y(2)$

$y'(2) = y(3)$

$y'(3) = y(4)$

$$y'(4) = \frac{1}{\beta y1}(y4 + (1+\lambda)(y1y3 - y2^2) + \beta y3^2) \qquad (18)$$

$y'(5) = y(6)$

$$y'(6) = -\frac{Pr}{\left(1+\frac{4}{3R}\right)}(y1y6 - my2y5 + \gamma y5) \qquad (19)$$

The governed equations are solved numerically using MATLAB using bvp4c

1.3. Results and Discursion

The Following Graphs Gives the variation of velocity, temperature and concentration

Fig 2 shows the effect of Pr Prandtl number on temperature, it shows that the temperature decreases with the increase of Pr. Physically it can be stated as the reduction in temperature is due to the thermal diffusivity, as thermal diffusivity decreases the Pr number increases and thus the temperature decreases.

Fig 3 shows the effect of ratio of relaxation and retardation time λ on velocity, increase in λ cause the reduction of boundary layer velocity of fluid.

fig 4 shows the effect of heat source parameter $\lambda_1$on temperature.it is clear that heat source gives an increase in the temperature of the fluid, physically the increase of heat source in the boundary layer generates energy which causes the temperature of the fluid to increase.

fig 5 and fig 6 shows the effect of Deborah number β on the fluid velocity, as β increases the velocity increases, physically Deborah number β is proportional to the rate of stretching sheet, the increase of β results in a higher fluid motion in the boundary layer.

fig 7 it shows the effect of R on temperature profile, as temperature distribution increases with the increase in the value of R, this is due to fact that the thermal boundary layer thickness increases with an increase in thermal radiation, thus to proceed cooling process faster the radiation should be minimized.

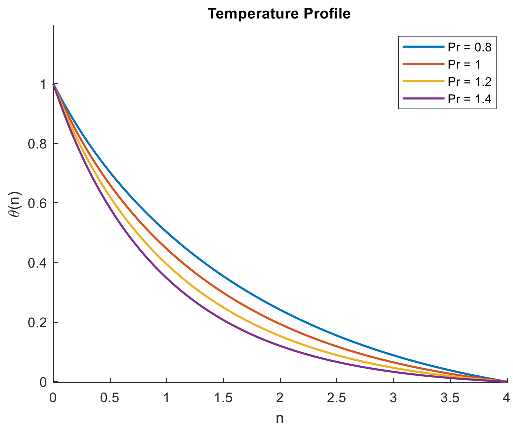

Figure 2: Temperature profile for Pr

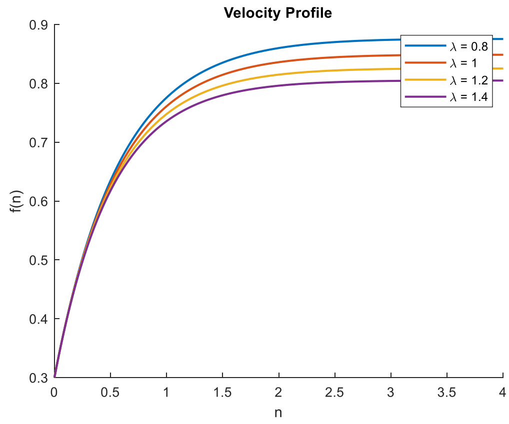

Figure 3: velocity profile for λ

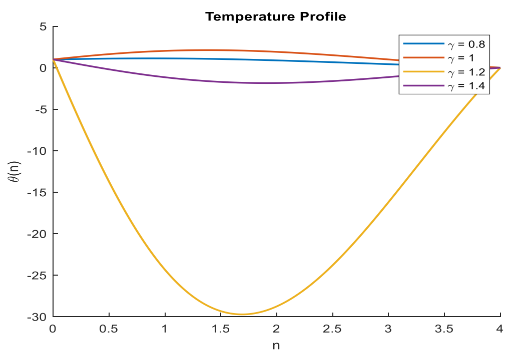

Figure 4: temperature profile for γ

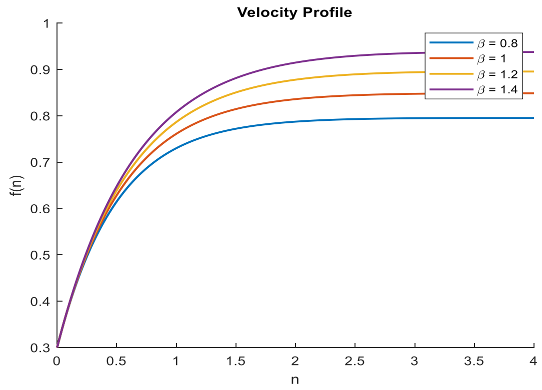

Figure 5: velocity profile for β

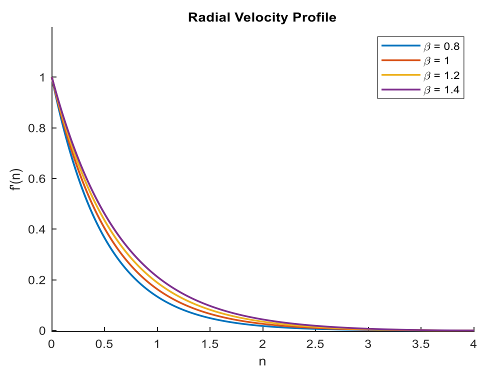

Figure 6: Radial velocity profile for β

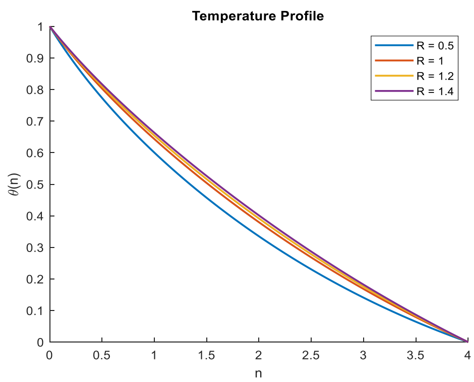

Figure 7: Temperature profile for R

Nomenclature

$A_1 A_2$ constants

C concentration [k mole/m^3]

c_p specific heat at constant pressure

C_∞ species concentration far away from wall

$C_w$species concentration at the wall

D diffusion coefficient[m^2/s]

E extra stress tensor

R Radiation parameter

R_1 Rivlin-Erickse tensor

U_w shrinking velocity [m/s]

u, v velocity components in the x, y directions, resp.[m/s]

λ_1 ratio of relaxation and retardation time

λ_2 relaxation time

τ Cauchy stress tensor

υ kinematic viscosity [m^2/s]

ρ fluid density [Kg/m]

T fluid temperature

T_∞temperature far away from the wall[K]

K fluid thermal conductivity [W/m/K]

K_s Rosseland mean absorption coefficient

Kr^* chemical reaction parameter

σ^* Stefan-Boltzmann constant

θ non dimensional temperature

ϕ non dimensional concentration

q_r radiative heat flux

Pr Prandtl number

β Deborah number

Sc Schmidt number

γ heat source parameter

x distance along the wall[m]

y distance normal to the wall [m]

ξ similarity variable

l characteristic length

m surface temperature parameter

Subscripts

W sheet surface

∞ infinity

Superscript

$'$ differentiation with respect to ξ

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