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Scroll Wave Unpinning with External Field is Impeded by Filament Alignment with Field

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Abstract

Scroll waves in excitable media can anchor to non-excitable inclusions. We study in a generic three-dimensional medium how external fields can unpin anchored scroll waves and destroy them. We find that external fields of increasing amplitude progressively deform the scroll wave's center of rotation (filament). Sufficiently large field force both segments of the anchored filament to align with the field. For smaller inclusions, the two segments of the filament get so close to each other that they collapse and the scroll wave is destroyed. For larger inclusions, the two segments stabilize at a distance that allows them to coexist and they are both oriented in the direction of the external field. In this configuration, they are insensitive to the presence to external fields and the scroll wave cannot be unpinned. The existence of such a stable scroll wave configuration may pose problems for defibrillation approaches that are based on unpinning.

Keywords: Type your keywords here, separated by semicolons: Excitable media, scroll wave, anchoring, unpinning, arrhythmia mechanisms

1. Introduction

Excitable media encompass a broad class of highly nonlinear, distributed non-equilibrium systems of physical, chemical, and biological systems [1], including nerve and cardiac tissues [2].

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Many three-dimensional excitable media allow the existence of self-sustained, rotating waves called scroll waves [3]. The most relevant such medium is cardiac tissue, in which scroll waves present as life-threatening arrhythmias [2]. Because of their medical relevance, substantial effort has been directed at understanding the dynamics of scroll waves and at devising methods to eliminate them.

The stability of scroll waves increases dramatically if the attach to heterogeneities in the medium in a process called anchoring [2,4,7]. Unanchored, the scroll wave drifts through the medium and may be destroyed spontaneously by several mechanisms. Once anchored, the scroll wave rotates stably and can only be destroyed if it is first unpinned (removed from the heterogeneity, [8-10] or more drastic interventions are taken (such as activation of most of the medium during defibrillation).

Recent studies have shown that filaments are repelled by unexcitable inclusions, but that this repulsion can be overcome and filaments can anchor if the external field is sufficiently strong [11]. In the present paper, we demonstrate the mechanism by which unpinning takes place in 3D and estimate the necessary external field amplitude for unpinning scroll wave filaments (compared to the amplitude needed to anchor them).

2. Materials and Methods

Numerical simulations were carried out in a generic reaction-diffusion model of an excitable medium with Barkley kinetics [12]:

$$\epsilon\partial\_t\, u = u(1-u)(u-(v+b)/a) + \nabla\ \cdot D\nabla \mathrm{u}$$
$$\partial_t v = u - v,$$

where u is the activator variable and$v$ the inhibitor variable. The external field is introduced by making the diffusion tensor Dspace dependent, $D(\boldsymbol{x}) = D_0 + \boldsymbol{E}\cdot\boldsymbol{x}$ the diffusivity tensor, so that$\boldsymbol{E}$ acts as an external field. Without loss of generality, we assumed that $\boldsymbol{E}$ is oriented along the x-axis. No-flux boundary conditions were set both at the border of the inclusion and at external medium boundaries. The values of the kinetic parameters a, b, and $\epsilon$are given in the caption of Fig.1; they were chosen such that in an unperturbed system ($\boldsymbol{E} = 0$), the spiral rotation was stationary and scroll wave filaments had positive tension [13].We induced spiral and scroll waves as previously described [14].

3. Results

For small fields ($\boldsymbol{E} = 0.01$), filaments that drift towards an unexcitable inclusion are deflected by it. Figure 1 shows how a filament in such a small field drifts towards in unexcitable inclusion, is deflected by the inclusion, and continues to drift in its original direction after it has passed the inclusion. This result is consistent with our previous report of scroll wave repulsion by an unexcitable inclusion [11].

When the external field is increased, the filament overcomes the repulsion and anchors to the inclusion. Figure 2 shows how the filament attaches and how the ends of the filament subsequently move in the direction of the external field $\boldsymbol{E}$. The stationary state of the filament is the result of a) the drift that pulls the filament in the direction of $\boldsymbol{E}$ and b) the filaments tension, i.e. its intrinsic tendency to shorten and therefore become straighter.Further increase of the external field leads a stationary filament that is more strongly curved (see Fig. 2B).

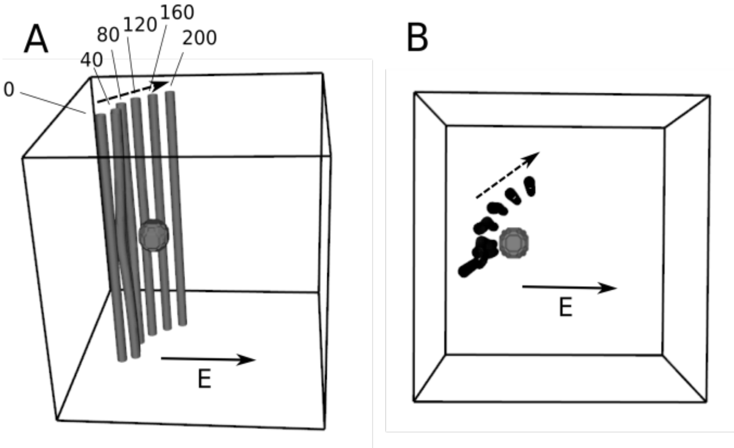

Figure 1. Filament deflection for small fields ($\boldsymbol{E} = 0.01$). **A:** Side view. Rod-shaped objects represent the filament at different moments of time, all superimposed in one Panel to illustrate the motion of the filament. Labels from “0” to “200” indicate the time at which each position is reached, in units of scroll wave rotations. The spherical object in front of the filaments is the inclusion. The dashed arrow indicates the drift direction, the solid arrow the direction of the external field. **B:** Top view. Medium parameters were $\epsilon = 0.02$, $a = 0.9$, $b = 0.05$, $120 \times 120 \times 120$nodes,$\Delta x = 0.25$, $D = 0.5$.

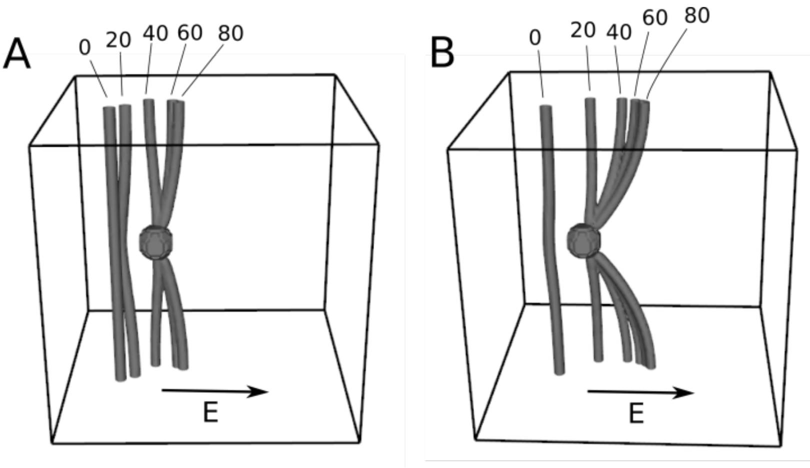

Figure 2. Filament anchoring and bending for intermediate fields ($\boldsymbol{E} = 0.02$ in Panel A, $\boldsymbol{E} = 0.04$ in Panel B). Notations as in Fig.1.

If the external field is sufficiently large, the free ends of the filament move so far to the side that they leave the upper and lower faces of our medium. Both segments of the filament straighten out and approach each other in the process. Eventually, the distance between the two segments becomes too small for the two associated scroll waves to coexist. This situation is analogous to pushing two counter-rotating spirals closer and closer together;

eventually there will not be enough space between the filaments for the waves to pass through and both filament segments collapse, starting from the inclusion. Figure 3 illustrates this process. Once the filament has detached (see the filament snapshot labeled "182"), its tension quickly pulls it towards the boundary of the medium on which the filament ends are located, and the filament disappears at $t = 190$ (measured in spiral periods).

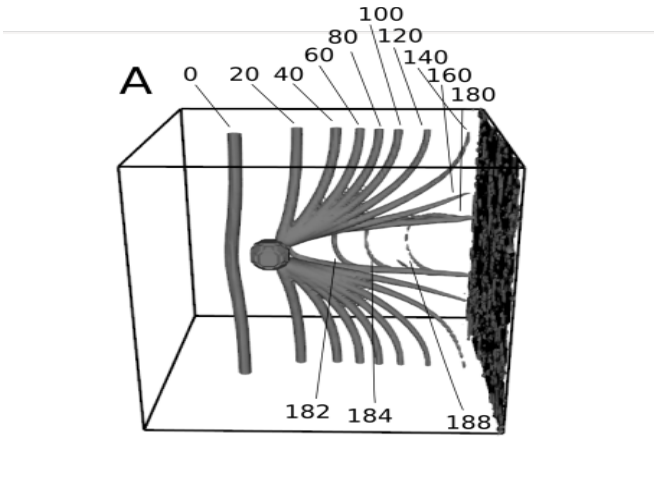

Figure 3. Filament unpinning for large fields (***E***=0.05). **A:** Side view. Note that the last filament snapshots (182, .., 188) have been spaced more closely in time to better illustrate the process of unpinning.

Our most interesting finding is that the mechanism of unpinning described in the previous paragraph fails for sufficiently large inclusions. If the inclusion is sufficiently large, the two filament segments can settle in a configuration in which they are both completely aligned with the external field, yet separated far enough to allow for their coexistence (see Fig. 4). Since the filament segments are then aligned with the external field, the field no longer exerts a force on them, and it will not be possible to unpin such filaments with any strength of field. This may have important consequences for defibrillation strategies in the heart that are based on unpinning: If scroll waves are anchored to sufficiently large heterogeneities, it may be impossible to unpin them with external fields.

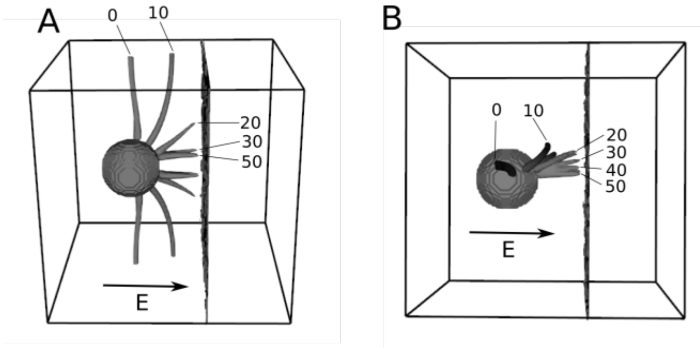

Figure 4. For large anchors ($r = 4$) and large fields ($\boldsymbol{E} = 0.1$), filaments align with the field and cannot be unpinned. The plane just right of the center of the medium indicates the boundary between medium in which waves can propagate (to the left), and medium in which low diffusion (due to the gradient in diffusion) makes wave propagation impossible. **A:** Side view. **B:** Top view.

The interaction of drifting filaments with unexcitable inclusions can thus be summarized as follows: Below a certain threshold drift speed, the filament cannot overcome the repulsion of the inclusion and is deflected by it. Sufficient drift speed will cause the filament to anchor. As long as the drift speed is below a second threshold, the filament stays anchored and is only more and more deformed as the external field is increased. Above the second threshold, the filament fully aligns with the external field. If the inclusion is sufficiently large, this leads to a stable configuration of the two filament segments that is no longer sensitive to external fields. If the inclusion is so small that two attached, parallel filament segments cannot coexist and the filament collapses.

Table 1. Anchoring and unpinning for different field strengths and inclusion sizes.

	$\boldsymbol{E}$=0.005	$\boldsymbol{E}$=0.01	$\boldsymbol{E}$=0.02	$\boldsymbol{E}$=0.04	$\boldsymbol{E}$=0.05	$\boldsymbol{E}$=0.1
$r = 1$	Not anchored	Anchored	Anchored	Anchored	Unpinned	Unpinned
$r = 2$	Not anchored	Not anchored	Anchored	Anchored	Unpinned	Unpinned
$r = 4$	Not anchored	Not anchored	Not anchored	Anchored	Anchored	Anchored

Table 1 summarizes our anchoring and unpinning results for three different inclusion sizes. For r=1, attachment occurs around 0.075, while detachment occurs around 0.045; for r=2, attachment occurs around 0.15, while detachment occurs around 0.045; for$r = 4$, attachment occurs around 0.3 and there is no detachment..

4. Discussion

We have shown the detailed 3D dynamics of anchoring of filaments to unexcitable inclusions and how they assume their stationary shape. We have also shown that for sufficiently strong external fields, the anchored filament becomes unstable and detaches from the inclusion if the inclusion is sufficiently small, but that

The results presented here are fully consistent with earlier work, which was directed at quantifying the forces the filament experiences when it approaches an unexcitable obstacle [11]. They are also consistent with a different study that determined the shape of a filament that is anchored at both ends and exposed to an external field [15]. In that paper, the filament assumed the shape of a hyperbolic cosine, in observance of the geodesic principle. In our current manuscript, two branches of the filament go from the inclusion to opposite directions, but each branch again has the shape of a hyperbolic cosine as the geodesic principle prescribes.

5. Conclusions

External fields are only effective for unpinning scroll waves if they are anchored to sufficiently small inclusions. For larger inclusions, the filament segments can align with the external field so that they are no longer affected by it.

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