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Using Analogical Reasoning in Teaching Mathematics: A Survey of Mathematics Teachers at Secondary Schools in the Mekong Delta – Vietnam

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Abstract

Teaching with analogies is an effective teaching strategy. Therefore, teachers from different countries have been studied and applied this strategy to teaching specific subjects in secondary schools. The study analyzed 20 lessons in the chapter "Coordinate Method in space" (Geometry 12) which were given by 18 mathematics teachers at the secondary schools in Mekong Delta region (Vietnam), and the results showed that only 5 lessons in which teachers used analogical reasoning to help students construct new knowledge.

Keywords: Analogical reasoning; analogy; teaching with analogy; co-ordinate methods in space; mathematics education, TWA model

1. Introduction

Currently, the use of analogies in teaching mathematics and sciences was interested in by many educators from different countries. It is not to only helps students review previous knowledge but also promote active activities of students during their learning new knowledge.

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In teaching mathematics in schools, according to [1], analogical reasoning has many applications as building meanings of new knowledge, building hypotheses to discover new knowledge. However, the issue was whether mathematics teachers prefer to the application of analogies in their teaching or not. In order to make clear the above issue, we conducted an investigation with following two questions:

1. *Do mathematics teachers of secondary schools prefer to choose analogical reasoning as an effective teaching strategy?*
2. *In the cases of using analogies in teaching, how do they use analogies in their teaching?*

2. Background

Conception of analogy

According to Visual Mathematics Dictionary, analogy is "a type of reasoning in which it is assumed that if there is a similarity or sameness between two problems or methods in some aspects, they may be alike in other aspects. This is not a reliable reasoning because it is not always true" [2].

The Random House College Dictionary [3] defined an analogy as "a partial similarity between like features of things, on which a comparison may be based", and according to Logic, analogy is "a form of reasoning in which one thing is inferred to be similar to another thing in a certain respect on the basis of the known similarity in other respects".

Gentner described an analogy as a correspondence of knowledge from a domain (base) to a target. So, we want to explain some new concepts (target) to mention some of the concepts already known or understood (base). It means we consider similar relationships between a base and a target [4].

Application of the analogy in teaching

According to [5], an analogy is a reasoning based on the similarity of the nature and relationship of the various mathematical objects. The analogy is intuitive and easy to understand, so it is often used in teaching mathematics. But it should be noted that the analogy may lead to wrong conclusions.

Harsha Patil and Ramchandra Tiwari showed that the advantages of analogies in teaching are visualization process, real world linkage, motivational function, encouraging the teacher to consider the students' prior knowledge, and two out of the disadvantages of analogies are analog unfamiliarity (causing great confusion and misunderstanding if it is used), incorrect transfer of attributes [6].

In order to help teachers to use analogies in teaching sciences in an effective way, Glynn introduced the model "TWA" (Teaching With Analogy). The TWA model consists of six steps as follows [7]:

Step 1: Introduce the target concept;

Step 2: Review the analog concept;

Step 3: Identify relevant features of the target and analog;

Step 4: Map similarities;

Step 5: Indicate where the analogy breaks down; and

Step 6: Draw conclusions.

Basing on the TWA model, Loc and Uyen [8] created the rubric for evaluating the level of using analogical reasoning in teaching mathematics (*see Table 4 below*). Besides, the authors of [9] showed that "for teachers, TWA model also reveals some difficulties of its own such as: selecting appropriate analog and establishing the correspondences between the analog and the target. To overcome this problem, when using the model, the teacher needs to master the characteristics of analog knowledge and target knowledge, and to select appropriate analog".

3. Methodology

- *Teacher surveyed:* 18 mathematics teachers who are teaching in different provinces in Mekong Delta, Vietnam.
- *Class observation:* we observed 20 lessons which were given by 18 teachers surveyed. *(see Table 1)*
- *Mathematics content focus:* We focused on lessons in Chapter "Coordinate Methods in space" (Geometry 12) because the structure of this chapter was similar to the one of Chapter "Coordinate methods in plane" which students have learned in Geometry 10 before. Therefore, while teaching Chapter "Coordinate Methods in space" (Geometry 12), teachers find it not difficult to use analogies to teach mathematical contents in this chapter. *(see Table 1)*

Table 1: Teachers and their lesson observed

	Teacher	**Secondary School**	**Lesson observed**	**Date of giving the lesson** *(day/month/year)*
1	T. C. L	The An Phu secondary school, An Giang province	The parametric equation of a straight line in space	11/02/2014
2	N. H. T.	The Nguyen Van Thoai secondary school, An Giang province	The parametric equation of a straight line in space	24/02/2014
3	V. C. L.	The Nguyen Van Thoai secondary school, An Giang province	Practice of solving problems on the equation of a sphere	15/05/2014
4	D. H. N.	The Nguyen Van Thoai secondary school, An Giang province	Practice of solving problems on the equation	17/05/2014

			of a straight line	
5	P. T. K. H.	The Nguyen Thi Minh Khai secondary school, Ben Tre province	The general equation of a plane	21/02/2014
			The distance from a point to a plane	23/02/2014
6	L. P. T. K.	The Nguyen Thi Đinh secondary school, Ben Tre province	The parametric equation of a straight line in space	03/02/2014
7	N. H. D.	The Nguyen Thi Đinh secondary school, Ben Tre province	The general equation of a plane	25/02/2014
8	L. X. M.	The Hoa An secondary school, Hau Giang province	The general equation of a plane	15/02/2014
9	L. M. N.	The Phu Dien secondary school, – Đong Thap province	The general equation of a plane	08/01/2014
			The distance from a point to a plane	16/01/2014
10	P. V. T.	The Nguyen Thong secondary school, Vinh Long province	The general equation of a plane	24/01/2014
11	V. V. T.	The Nguyen Thong secondary school, Vinh Long province	The parametric equation of a straight line in space	13/02/2014
12	N. T. T. A.	The Nguyen Thong secondary school, Vinh Long province	The relative positions of two straight lines in space	01/03/2014
13	N. N. N.	The Phu Thinh secondary school, Vinh Long province	Practice of solving problems on the general equation of a plane	16/01/2014
14	T. T. L. H.	The Long Phu secondary school, Vinh Long province	Practice of solving problems on the general equation of a plane	13/01/2014
15	N. T. K. C.	The Trung Vuong, Vinh Long province	Condition of two parallel planes	15/01/2014
16	V. V. N.	Trường THPT Vo Van Kiet secondary school, Vinh Long province	Practice of solving problems on the equation of a straight line	19/03/2014
17	H. Q. T.	Trường THPT Vo Van Kiet secondary school, Vinh Long province	Practice of solving problems on the general equation of a plane	19/03/2014
18	L. T. B. T.	Trường THPT Vo Van Kiet secondary school, Vinh Long province	The parametric equation of a straight line in space	22/03/2014

In this study, we did not make any impacts to the teaching of teachers. In these lessons, the teachers chose their own teaching strategies. Each lesson was observed and recorded in detail by other mathematics teacher who was working in the secondary schools according to the record form that we recommended as follows *(see Table 2 and Table 3):*

Table 2: Record form of class observation (in Vietnamese)

BIÊN BẢN DỰ GIỜ

Trường: THPT……

Người dạy: …… Ngày:……

Tên bài dạy:…… Lớp:……

Người dự:……

Nội dung bài học	Hoạt động GV	Hoạt động HS	Nhận xét
……	……	……	……
……	……	……	……
……	……	……	……
……	……	……	……
……	……	……	……
……	……	……	……
……	……	……	……
……	……	……	……
……	……	……	……
……	……	……	……

Table 3: Record form of class observation (translated from Vietnamese record form)

CLASS OBSERVATION RECORD

The secondary school:…….

Teacher:……..Date:…….

The name of lesson: ……Class:……

The observer:……

Content of lesson	Activities of teacher	Activities of students	Remarks
.....	.....	.....	.....
.....	.....	.....	.....
.....	.....	.....	.....

- *Evaluating the level of using analogies in the classroom of teachers*

In order to evaluate the level of using analogies in the classroom of teachers, we used the rubric for evaluating the use of analogies which was suggested by Loc and Uyen (*see Table 4)* [8].

Table 4: Rubric for evaluating teachers' the use of analogies [8]

Mark	Levels of using analogies in teaching
0	Not using any anlogogy.
1	Only talking about analog
2	Recalling characteristics of analog, but not establishing any correspondence between analog and target
3	Establishing correspondences between analog and target
4	Drawing a conclusion about the analogy and comparison of the new material with the already learned material

- *Survey time:*The investigation was implemented in 2014.

4. Results and disscusion

After analyzing the 20 class – observation records, we found that 5 out of 20 lessons in which teachers used analogical reasoning to help students learn new knowledge. *(see Table 5)*

Table 5: Lessons and teachers' the use of analogies

Lesson	The number of teacher giving the lesson	The number of lesson in which analogies used	Average mark of the level of using analogy
Practice of solving problems on the equation of a sphere	1	0	0
The general equation of a plane	4	1	0.25
Conditions of two parallel planes	1	0	0
The distance from a point to a plane	3	3	2.33
Practice of solving problems on the general equation of a plane	3	0	0
The parametric equation of a straight line in space	5	1	0.2
The relative positions of two straight lines	2	0	0
Practice of solving problems on the equation of a straight line in space	2	0	0

The result showed that many mathematics teachers did not choose analogies to help students learn new knowledge: there were a few lessons (5 of 20 lessons) in which teachers used analogy as a teaching strategy.

Mathematics contents taught with analogical reasoning were the general equation of a plane, the parametric equation of a straight line in space, and especially formula of distance from a point to a plane which there were all of three teachers preferring to use analogies to their teaching.

In order to know more about how teachers used analogies in their teaching process, we analyze three lectures in which teachers made use of analogical reasoning as a teaching tool as follows*:*

Teaching activities of teacher L. X. M. *(see Table 6)*

Table 6: How to use analogy to teaching of L. X. M. (*quoted from class observation record)*

Mathematical content	Activities of teacher	Activities of students
Definition: In Oxyz, an equation in form : $Ax+By+Cz+D=0$, where A, B, C are not simultaneously equal to 0, is called the general equation of a plane.	- Ask students to restate the general equation of a straight line in plane Oxy - Introduce: In space Oxyz, analogically, we have the general equation of a plane, and - Ask students to read how to construct the general equation of a plane in the textbook.	- Answer: The general equation of a line: $Ax+By+C=0$ - Read how to construct the general equation of a plane in the textbook

Comment:

Table 6 showed that L. X. M. used "the general equation of a straight line in plane Oxy" as analog to teach students target knowledge "the general equation of a plane in space Oxyz", but she gave no links between the analog and the target.

Teaching activities of teacher L. M. N. *(see Table 7)*

Table 7: How to use analogy to teaching of L. M. N. *(quoted from class observation record)*

Mathematical content	Activities of teacher	Activities of students
Theorem: In space Oxyz, given a plane (α) which its equation is:	- Ask students to restate the formula for computing the distance	- Answer: The distance from $M_0(x_0;y_0)$ to $\Delta: Ax+By+C=0$ is : $d(M_0,\Delta)=$

$Ax+By+Cz+D=0$ and a point $M_0(x_0;y_0;z_0)$. The distance from a point M_0 to a plane (α), denoted by $d(M_0,(\alpha))$, is calculated by the following formula : $d(M_0,(\alpha)) = \frac{\lvert Ax_0+By_0+Cz_0+D\rvert}{\sqrt{A^2+B^2+C^2}}$	from a point to a straight line in plane Oxy - Ask: In a similiar way, please predict the formula for computing the distance from $M_0(x_0;y_0;z_0)$ to a plane (α) - Confirm the formula (1) - Ask students to read how to construct the formula in the textbook	$\frac{\lvert Ax_0+By_0+C\rvert}{\sqrt{A^2+B^2}}$ - Answer: $d(M_0,(\alpha)) = \frac{\lvert Ax_0+By_0+Cz_0+D\rvert}{\sqrt{A^2+B^2+C^2}}$ (1) - Read how to construct the formula in the textbook

Comment:

Through Table 7, we found that at first, teacher L. M. N. asked students to restate the formula for computing the distance from a point to a line in the plane Oxy, which they already learned in Geometry 10; then he suggested students to predict what the formula for computing the distance from a point to a plane in space Oxyz is. Finally, he guided students to read textbooks. The above teaching activities of the teacher L. M. N. were to help students foresee new knowledge from old knowledge by analogical reasoning.

Teaching activities of teacher P. T. K. H. *(see Table 8)*

Table 8: How to use analogy to teaching of P. T. K. H. *(quoted from class observation records)*

Mathematical content	Activities of teacher	Activities of students
Theorem: In space Oxyz, given a plane (α) whose equation is: $Ax+By+Cz+D=0$ and a point $M_0(x_0;y_0;z_0)$. The distance from a point M_0 to a plane (α), denoted by $d(M_0,(\alpha))$, is calculated by the following formlula :	- Given a point M and a plane (α), according to Geometry 11, how can we compute the distance from M to (α)? - In space Oxyz, How do we can compute $\lvert\overrightarrow{M_0M_1}\rvert$, where M ($x_0$, y_0, z_0) and (α) :Ax+By+Cz+D =	- Find the perpendicular projection M_1 of M_0 on (α), and calculate the length of M_0M_1. $\lvert\overrightarrow{M_0M_1}\rvert = \frac{\lvert\overrightarrow{M_0M_1}.\vec{n}\rvert}{\lvert\vec{n}\rvert}$

$d(M_0,(\alpha)) = \frac{\lvert Ax_0 + By_0 + Cz_0 + D\rvert}{\sqrt{A^2 + B^2 + C^2}}$	0 ? - Determine that (1) is the formula for computing the distance from a point M (x_0, y_0, z_0) to plane: Ax+By+Cz+D = 0.	$\frac{\lvert -Ax_0 - By_0 - Cz_0 + Ax_1 + By_1 + Cz_1\rvert}{\sqrt{A^2 + B^2 + C^2}} = \frac{\lvert Ax_0 + By_0 + Cz_0 + D\rvert}{\sqrt{A^2 + B^2 + C^2}}$ (1) - Take note

Comment

Table 8 showed that unlike the choice of the analog of L. M. N, the teacher P. T. K. H. chose the process of computing the distance from a point to a plane that students learned in Geometry 11 as analog. Then, she guided her students how to find the formula for calculating the distance from a point to a plane. It was clear that teaching activities of the teacher P. T .K. H. also promoted learning activities of the students in her teaching process.

5. Limitations of the study

The results of the study reflected only partly the status of the using analogies in teaching mathematics in secondary schools of the Mekong Delta region – Vietnam because of the below limitations:

- *Limitation of subject surveyed:* Recently, in the Mekong Delta there are 13 different provinces. Because of finance, we observed classrooms of teachers from only 5 provinces: An Giang, Vinh Long, Hau Giang, Đong Thap and Ben Tre.

- *Limitations of research procedures:* The results of the study could be more convinced if we used mixed methods instead of only observation method.

6. Conclusion

Through the above results, we could recognize that there were a few mathematics teachers selecting analogical reasoning as a teaching strategy even though for mathematics contents of Chapter "Coordinate Methods in Space", it was not only easy to choose appropriate analog but there were some different sources suitable to choose. This was a problem that teacher - training universities need to pay much attention to training their students how to apply teaching with analogy and need to show them high the effectiveness of this strategy.

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