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New Arithmetic Triangular Fuzzy Number for Solving Fully Fuzzy Linear System using Inverse Matrix

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Abstract

This paper present a new concept arithmetic of triangular fuzzy number, namely by using a board area concept of triangular fuzzy number, so that we will get a form multiplication of fuzzy numbers in some cases. This new arithmetic concept will be applied for solve the fully fuzzy linear system using inverse matrix. Furthermore, to illustration will give numerical examples of solving fully fuzzy linear system using inverse matrix with a case of multiplication positive fuzzy number and negative fuzzy number.

Keywords: triangular fuzzy number; arithmetic fuzzy number; fully fuzzy linear system 2010 Mathematics Subject Classification: 94D05; 08A72; 15B15.

1. Introduction

Fuzzy is the first time introduced by Lotfi A Zadeh which associates fuzzy sets with a function that states the degree of suitability of element in accordance with the concept which requirement for membership of a fuzzy set. This function is called the membership function and the function value is called the degree of membership of an element in the fuzzy set [14]. Fuzzy application in algebra one of them is a fuzzy linear system, fuzzy linear systems are in the form of $\boldsymbol{A} \otimes \tilde{\boldsymbol{x}} = \tilde{\boldsymbol{b}}$, with $\boldsymbol{A}$ is a real matrix and $\tilde{\boldsymbol{x}}$, $\tilde{\boldsymbol{b}}$ are fuzzy vector. If matrix $\boldsymbol{A}$ is a fuzzy matrix, the linear systems is called a fully fuzzy linear system which can be written in the form $\tilde{\boldsymbol{A}} \otimes \tilde{\boldsymbol{x}} = \tilde{\boldsymbol{b}}$. Beside that there is also fuzzy linear system in the form of $\tilde{\boldsymbol{A}} \otimes \tilde{\boldsymbol{x}} \oplus \tilde{\boldsymbol{c}} = \tilde{\boldsymbol{B}} \otimes \tilde{\boldsymbol{x}} \oplus \tilde{\boldsymbol{d}}$ called the dual fully fuzzy linear system discuss by [6,8,9,10,12 and 13].

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Fully fuzzy linear system have been widely discussed including [1,2,4,5] with various methods and algebraic of fuzzy number. But several of authors only give an example in case of multiplying positive fuzzy number so the solutions obtained are not compatible. In this article a new definition will be given to determine a fuzzy number that is said to be positive or negative using comparison of representation areas triangular fuzzy numbers based on the r axis. This new definition causes multiplication operation to be divided into four case and the results obtained are compatible, for more details given example solving fully fuzzy linear system based on new definitions and multiplication case on positive fuzzy number and negative fuzzy number using inverse matrix.

2. Preliminaries and Basic Definition

In Fuzzy terms are also known as fuzzy sets and fuzzy number, here are given definition of fuzzy sets by lotfi A. Zadeh [14] and zimmermann [16].

Definition 2.1: A fuzzy set $\widetilde{M}$ in X is a characterized by membership function $f_{\widetilde{M}}(x)$ which associates with each points in X real number in the interval $[0,1]$, with the value of $f_{\widetilde{M}}(x)$ at x representing the "grade of membership" of x in $\widetilde{M}$.

Definition 2.2: If X is a collection of objects that are denoted in general by x, the a fuzzy set $\widetilde{M}$ in X is a sequential set of pairs $\widetilde{M} = (X, \mu_{\tilde{A}}(x) | x \epsilon X)$ with $\mu_{\widetilde{M}}$ is a membership function of the fuzzy set $\widetilde{M}$ which a mapping of the universal set X in the interval [0,1].

Some basic definition and theories related to fuzzy number has been discussed by [1,3,4,5 and 10].

Definition 2.3: A fuzzy number is a fuzzy set $\tilde{u}: R \to [0,1]$ which satisfies

1. $\tilde{u}$ is upper semi continuous;
2. $\tilde{u}(x) = 0$ outside some interval $[c, d]$;
3. There exist real number $[a, b]$ in $[c, d]$ such that

   (i) $\tilde{u}(x)$ monotonic increasing in $[c, a]$;
   (ii) $\tilde{u}(x)$monotonic decreasing in $[b, d]$;
   (iii) $\tilde{u}(x) = 1, for\ a \leq x \leq b$.

Definition 2.4: A fuzzy number $\tilde{u}$ in R is pair $[\underline{u}(r), \overline{u}(r)]$, which satisfies the following:

1. $\underline{u}(r)$ is a bounded left continuous non decreasing function over [0,1];
2. $\overline{u}(r)$ is a bounded left continuous non increasing function over [0,1];
3. $\underline{u}(r) \leq \overline{u}(r), 0 \leq r \leq 1$.

The form of the function of the fuzzy triangular number $\tilde{u} = (a, \alpha, \beta)$ is:

$$\mu_{\tilde{u}} = \begin{cases} 1 - \dfrac{(a-x)}{\alpha} & a - \alpha \le x \le a \\ 1 - \dfrac{(a-x)}{\alpha} & a - \alpha \le x \le a \\ 0 & others. \end{cases}$$

The parametric form fuzzy number $\tilde{u} = [\underline{u}(r), \overline{u}(r)]$ can be represented as:

$$\underline{u}(r) = a - (1-r)\alpha \qquad (2.1)$$

$$\overline{u}(r) = a + (1-r)\beta \qquad (2.2)$$

3. New Concept Arithmetic for Triangular Fuzzy Number

Triangular fuzzy number is $\tilde{u} = (a, \alpha, \beta)$ where a is midpoint, α is the width to the left and β is the width to the right. Next, a new definition will be given to determine positive triangular fuzzy number and negative triangular fuzzy number based on the comparison of representation areas triangular fuzzy number located on the right and left r axis.

Triangular fuzzy number $\tilde{u} = (a, \alpha, \beta)$ is defined positive and negative based on case following:

a. If $a - \alpha \ge 0$, then it is clear that $\tilde{u}$ is positive, if $a + \beta \le 0$, then it is clear that $\tilde{u}$ is negative. Seen in the picture below:

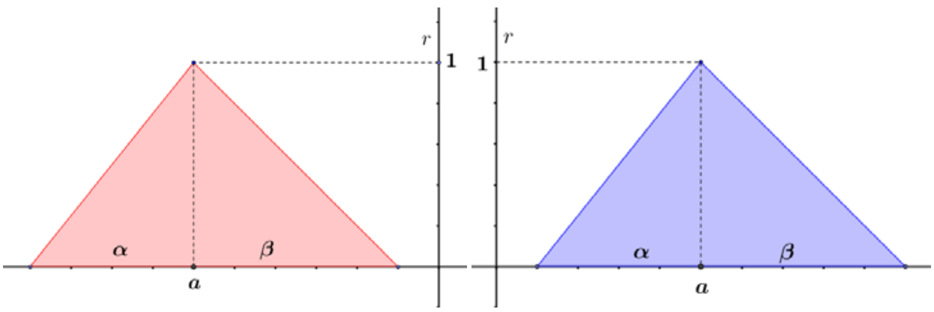

Figure 1: Triangular fuzzy number $\tilde{u} = (a, \alpha, \beta)$ negative and positive

b. For $a > 0$ and $a - \alpha < 0$, Seen in the picture below:

$$R = \text{ } area\ to\ the\ right\ r - axis$$
$$L = \text{ } area\ to\ the\ left\ r - axis$$

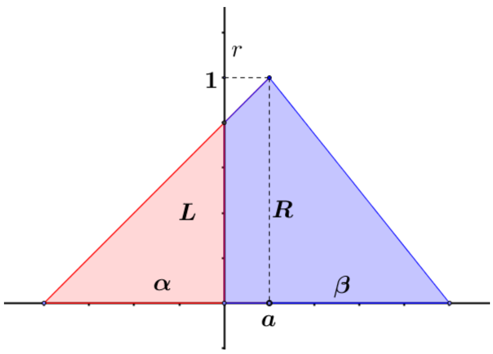

Figure 2: Triangular fuzzy number $\tilde{u} = (a, \alpha, \beta)$ with $a > 0$

$\tilde{u} = (a, \alpha, \beta)$ said positive if $R - L > 0$ or can be written as $\frac{\beta-\alpha}{2} + 2a - \frac{a^2}{\beta} > 0$, and $\tilde{u} = (a, \alpha, \beta)$ said negative if $R - L < 0$ or can be written as $\frac{\beta-\alpha}{2} + 2a - \frac{a^2}{\beta} < 0$.

c. For $a < 0$ and $a + \beta > 0$, Seen in the picture below:

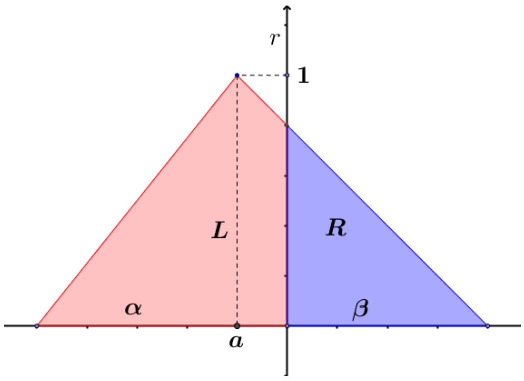

Figure 3: Triangular fuzzy number $\tilde{u} = (a, \alpha, \beta)$ with $a < 0$

$\tilde{u} = (a, \alpha, \beta)$ said positive if $R - L > 0$ or can be written as $\frac{\beta-\alpha}{2} + 2a + \frac{a^2}{\beta} > 0$, and $\tilde{u} = (a, \alpha, \beta)$ said negative if $R - L < 0$ or can be written as $\frac{\beta-\alpha}{2} + 2a + \frac{a^2}{\beta} < 0$.

d. if $a = 0$, Seen in the picture below:

$\tilde{u}$=(a,α,β) said positive if R-L>0 or can be written β-α>0, and $\tilde{u}$=(a,α,β) said negative if R-L<0 or can be written as β-α<0.

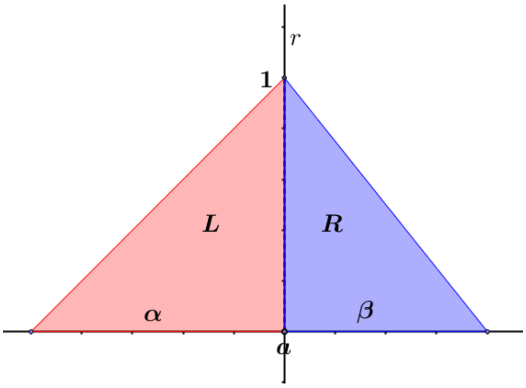

Figure 4: Triangular fuzzy number $\tilde{u} = (a, \alpha, \beta)$ with $a = 0$

The new arithmetic operation on triangular fuzzy number, let $\tilde{u} = (a, \alpha, \beta)$, $\tilde{v} = (b, \gamma, \delta)$ and k is a scalar, based on (2.1) and (2.2) then:

$$\tilde{u} = \left[\underline{u}(r), \overline{u}(r)\right] = [a - (1 - r)\alpha, a + (1 - r)\beta] \tag{3.1}$$

$$\tilde{v} = \left[\underline{v}(r), \overline{v}(r)\right] = [b - (1 - r)\gamma, b + (1 - r)\delta] \tag{3.1}$$

Next for algebraic process on two triangular fuzzy numbers consist of:

a. A
ddition

$$\tilde{u} \oplus \tilde{v} = \left[\underline{u}(r) + \underline{v}(r), \overline{u}(r) + \overline{v}(r)\right]$$

$$= [(a + b) - (1 - r)(\alpha + \gamma), (a + b) + (1 - r)(\beta + \delta)]$$

This parametric form can be transforming back into triangular form, we have

$$\tilde{u} \oplus \tilde{v} = (a + b, \alpha + \gamma, \beta + \delta)$$

b. Subtraction

$$\tilde{u} \ominus \tilde{v} = \left[\underline{u}(r) - \overline{v}(r), \overline{u}(r) - \underline{v}(r)\right]$$

$$= [(a - b) - (1 - r)(\alpha + \delta), (a - b) + (1 - r)(\beta + \gamma)]$$

This parametric form can be transforming back into triangular form, we have

$$\tilde{u} \ominus \tilde{v} = (a - b, \alpha + \delta, \beta + \gamma)$$

c. Scalar multiplication

$k \otimes \tilde{u} = k \otimes (a, \alpha, \beta)$

$$= \begin{cases} (ka, k\alpha, k\beta) & k \geq 0 \\ (ka, -k\alpha, -k\beta) & k < 0 \end{cases}$$

d. Multiplication

The applicable multiplication operation is $\tilde{w} = \tilde{u} \otimes \tilde{v} = [\underline{w}(r), \overline{w}(r)]$ for every $r \in [0,1]$. Here are some cases for triangular fuzzy number multiplication operation

(i) Case 1, if $\tilde{u}$ positive and $\tilde{v}$ positive, then:

$$\begin{cases} \underline{w}(r) = \underline{u}(r)\underline{v}(1) + \underline{u}(1)\underline{v}(r) - \underline{u}(1)\underline{v}(1) \\ \overline{w}(r) = \overline{u}(r)\overline{v}(1) + \overline{u}(1)\overline{v}(r) - \overline{u}(1)\overline{v}(1) \end{cases} \tag{3.3}$$

Substitution (3.1) and (3.2) to (3.3), we get

$\tilde{w} = [\underline{w}(r), \overline{w}(r)]$

$= [(a - (1 - r)\alpha)b + a(b - (1 - r)\gamma) - ab, (a + (1 - r)\beta)b + a(b + (1 - r)\delta) - ab]$

$$= [ab - (1 - r)(a\gamma + b\alpha), ab + (1 - r)(a\delta + b\beta)] \tag{3.4}$$

Let $\tilde{w} = (c, \xi, \psi)$ triangular fuzzy number with parametric form

$$\tilde{w} = [c - (1 - r)\xi, c + (1 - r)\psi] \tag{3.5}$$

Based on (3.4) and (3.5) obtained $c = ab, \quad \xi = a\gamma + b\alpha, \quad \psi = a\delta + b\beta$

So the multiplication operation for $\tilde{u}$ positive and $\tilde{v}$ positive can be transforming back into triangular form, we have:

$$\tilde{w} = \tilde{u} \otimes \tilde{v} = (ab, a\gamma + b\alpha, a\delta + b\beta) \tag{3.6}$$

(ii) Case 2, if $\tilde{u}$ positive and $\tilde{v}$ negative, then:

$$\begin{cases} \underline{w}(r) = \overline{u}(r)\underline{v}(1) + \overline{u}(1)\underline{v}(r) - \overline{u}(1)\underline{v}(1) \\ \overline{w}(r) = \underline{u}(r)\overline{v}(1) + \underline{u}(1)\overline{v}(r) - \underline{u}(1)\overline{v}(1) \end{cases} \tag{3.7}$$

Substitution (3.1) and (3.2) to (3.7), we have,

$\tilde{w} = [\underline{w}(r), \overline{w}(r)]$

$= [(a + (1 - r)\beta)b + a(b - (1 - r)\gamma) - ab, (a - (1 - r)\alpha)b + a(b + (1 - r)\delta) - ab]$

$$= [ab - (1-r)(a\gamma - b\beta), ab + (1-r)(a\delta - b\alpha)] \tag{3.8}$$

Based on (3.8) and (3.5) obtained: $c = ab,\ \ \xi = a\gamma + b\alpha,\ \ \psi = a\delta + b\beta$

So the multiplication operation for $\tilde{u}$ positive and $\tilde{v}$ negative can be transforming back into triangular form, we have:

$$\tilde{w} = \tilde{u} \otimes \tilde{v} = (ab, a\gamma - b\beta, a\delta - b\alpha) \tag{3.9}$$

(iii) Case 3, if $\tilde{u}$ negative and $\tilde{v}$ positive, then:

$$\begin{cases} \underline{w}(r) = \underline{u}(r)\overline{v}(1) + \underline{u}(1)\overline{v}(r) - \underline{u}(1)\overline{v}(1) \\ \overline{w}(r) = \overline{u}(r)\underline{v}(1) + \overline{u}(1)\underline{v}(r) - \overline{u}(1)\underline{v}(1) \end{cases} \tag{3.10}$$

Substitution (3.1) and (3.2) to (3.10) we have,

$$\tilde{w} = [\underline{w}(r), \overline{w}(r)]$$

$$= [(a - (1-r)\alpha)b + a(b + (1-r)\delta) - ab, (a + (1-r)\beta)b + a(b - (1-r)\gamma) - ab]$$

$$= [ab - (1-r)(-a\delta + b\alpha), ab + (1-r)(-a\gamma + b\beta)] \tag{3.11}$$

Based on (3.11) and (3.5) obtained: $c = ab,\ \ \xi = b\alpha - a\delta,\ \ \psi = b\beta - a\gamma$

So the multiplication operation for $\tilde{u}$ positive and $\tilde{v}$ negative can be transforming back into triangular form, we have:

$$\tilde{w} = \tilde{u} \otimes \tilde{v} = (ab, b\alpha - a\delta, b\beta - a\gamma) \tag{3.12}$$

(iv) C
ase 4, if $\tilde{u}$ negative and $\tilde{v}$ negative, then:

$$\begin{cases} \underline{w}(r) = \underline{u}(r)\overline{v}(1) + \underline{u}(1)\overline{v}(r) - \underline{u}(1)\overline{v}(1) \\ \overline{w}(r) = \overline{u}(r)\overline{v}(1) + \overline{u}(1)\overline{v}(r) - \overline{u}(1)\overline{v}(1) \end{cases} \tag{3.13}$$

Substitution (3.1) and (3.2) into (3.13) we have:
$$\tilde{w} = [\underline{w}(r), \overline{w}(r)]$$

$$= [(a + (1-r)\beta)b + a(b + (1-r)\delta) - ab, (a - (1-r)\alpha)b + a(b - (1-r)\gamma) - ab]$$

$$= [ab - (1-r)(-a\delta - b\beta), ab + (1-r)(-a\gamma - b\alpha)] \tag{3.14}$$

Based on (3.14) and (3.5) obtained: $c = ab,\ \ \xi = -a\delta - b\beta,\ \ \psi = -a\gamma - b\alpha$

So the multiplication operation for $\tilde{u}$ positive and $\tilde{v}$ negative can be transforming back into triangular form, we have:

$$\tilde{w} = \tilde{u} \otimes \tilde{v} = (ab, -a\delta - b\beta, -a\gamma - b\alpha) \tag{3.15}$$

e. Inverse

Before discuss inverses of triangular fuzzy numbers, it is first explained about the identity of triangular fuzzy number. A triangular fuzzy number $\tilde{1}$ is said to be an identity of triangular fuzzy number if $\tilde{1} = (1,0,0)$. Let $\tilde{u} = (a, \alpha, \beta)$ and $\tilde{v} = (b, \gamma, \delta)$, then $\tilde{u}$ have an inverse of multiplication operation if there is a fuzzy number $\tilde{v} = \frac{1}{\tilde{u}}$ such that $\tilde{u} \otimes \tilde{v} = \tilde{1}$. Because fuzzy number $\tilde{1} = (1,0,0)$ is a positive triangular fuzzy number, the multiplication operations consist of two multiplication case, namely:

(i) if $\tilde{u}$ positive and $\tilde{v}$ positive, then

$$\tilde{u} \otimes \tilde{v} = (a, \alpha, \beta) \otimes (b, \gamma, \delta) = (ab, a\gamma + b\alpha, a\delta + b\beta) = (1,0,0)$$

(ii) if $\tilde{u}$ negative and $\tilde{v}$ negative, then

$$\tilde{u} \otimes \tilde{v} = (a, \alpha, \beta) \otimes (b, \gamma, \delta) = (ab, -a\delta - b\beta, -a\gamma - b\alpha) = (1,0,0)$$

So that inverses of triangular fuzzy numbers are obtained with the condition $a \neq 0$ are:

$$\tilde{v} = \frac{1}{\tilde{u}} = \left(\frac{1}{a}, \frac{\alpha}{a^2}, \frac{\beta}{a^2}\right)$$

4. Solving Fully Fuzzy Linear System Using Inverse Matrix

General form of fully fuzzy linear system as below:

$$(\tilde{\boldsymbol{a}}_{\boldsymbol{11}} \otimes \tilde{\boldsymbol{x}}_{\boldsymbol{1}}) \oplus (\tilde{\boldsymbol{a}}_{\boldsymbol{12}} \otimes \tilde{\boldsymbol{x}}_{\boldsymbol{2}}) \oplus \dots \oplus (\tilde{\boldsymbol{a}}_{\boldsymbol{1n}} \otimes \tilde{\boldsymbol{x}}_{\boldsymbol{n}}) = \tilde{\boldsymbol{b}}_{\boldsymbol{1}}$$

$$(\tilde{\boldsymbol{a}}_{\boldsymbol{21}} \otimes \tilde{\boldsymbol{x}}_{\boldsymbol{1}}) \oplus (\tilde{\boldsymbol{a}}_{\boldsymbol{22}} \otimes \tilde{\boldsymbol{x}}_{\boldsymbol{2}}) \oplus \dots \oplus (\tilde{\boldsymbol{a}}_{\boldsymbol{2n}} \otimes \tilde{\boldsymbol{x}}_{\boldsymbol{n}}) = \tilde{\boldsymbol{b}}_{\boldsymbol{2}}$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$(\tilde{\boldsymbol{a}}_{\boldsymbol{n1}} \otimes \tilde{\boldsymbol{x}}_{\boldsymbol{1}}) \oplus (\tilde{\boldsymbol{a}}_{\boldsymbol{n2}} \otimes \tilde{\boldsymbol{x}}_{\boldsymbol{2}}) \oplus \dots \oplus (\tilde{\boldsymbol{a}}_{\boldsymbol{nn}} \otimes \tilde{\boldsymbol{x}}_{\boldsymbol{n}}) = \tilde{\boldsymbol{b}}_{\boldsymbol{n}}$$

The above fully fuzzy linear system can be made in the form a matrix $\tilde{\boldsymbol{A}} \otimes \tilde{\boldsymbol{x}} = \tilde{\boldsymbol{b}}$, with $\tilde{\boldsymbol{A}} = (\tilde{\boldsymbol{\alpha}}_{\boldsymbol{ij}}) = (\boldsymbol{a}_{\boldsymbol{ij}}, \boldsymbol{\alpha}_{\boldsymbol{ij}}, \boldsymbol{\beta}_{\boldsymbol{ij}})$ is a fuzzy matrix size nxn with a new notation $\tilde{\boldsymbol{A}} = (\boldsymbol{A}, \boldsymbol{M}, \boldsymbol{N})$, with $\boldsymbol{A} = (\boldsymbol{a}_{\boldsymbol{ij}}), \boldsymbol{M} = (\boldsymbol{\alpha}_{\boldsymbol{ij}})$ and $\boldsymbol{N} = (\boldsymbol{\beta}_{\boldsymbol{ij}})$ are three of matrix crisp nxn. While $\tilde{\boldsymbol{x}} = (\tilde{x}_1, \tilde{x}_2, \dots \tilde{x}_n)$ and $\tilde{\boldsymbol{b}} = (\tilde{b}_1, \tilde{b}_2, \dots \tilde{b}_n)$ are fuzzy vector which are each sized $nx1$. Next to get a solution on fully fuzzy linear system with $\tilde{\boldsymbol{A}} = (\boldsymbol{A}, \boldsymbol{M}, \boldsymbol{N})$, $\tilde{x} = (x, y, z)$

and $\tilde{b} = (b, g, h)$ we have:

$$(\boldsymbol{A}, \boldsymbol{M}, \boldsymbol{N}) \otimes (x, y, z) = (b, g, h)$$

Assume $\boldsymbol{A}$ is non-singular matrix and using arithmetic multiplication operation consist of 4 cases, then:

(i) Case 1, if $\widetilde{\boldsymbol{A}}$ positive and $\widetilde{\boldsymbol{b}}$ positive, then $\widetilde{\boldsymbol{x}}$ must also be positive

Based on the multiplication formula (3.6) then we have,

$$x = A^{-1}b$$

$$y = A^{-1}(g - Mx) \tag{4.1}$$

$$z = A^{-1}(h - Nx)$$

(ii) Case 2, if $\widetilde{\boldsymbol{A}}$ positive and $\widetilde{\boldsymbol{b}}$ negative, then $\widetilde{\boldsymbol{x}}$ must be negative

Based on the multiplication formula (3.9) then we have,

$$x = A^{-1}b$$

$$y = A^{-1}(g + Nx) \tag{4.2}$$

$$z = A^{-1}(h + Mx)$$

(iii) Case 3, if $\widetilde{\boldsymbol{A}}$ negative and $\widetilde{\boldsymbol{b}}$ positive, then $\widetilde{\boldsymbol{x}}$ must also be negative

Based on the multiplication formula (3.12) then we have,

$$x = A^{-1}b$$

$$y = A^{-1}(Nx - h) \tag{4.3}$$

$$z = A^{-1}(Mx - g)$$

(iv) Case 4, if $\widetilde{\boldsymbol{A}}$ negative and $\widetilde{\boldsymbol{b}}$ negative, then $\widetilde{\boldsymbol{x}}$ must also be positive

Based on the multiplication formula (3.12) then we have,

$$x = A^{-1}b$$

$$y = A^{-1}(-h - Mx) \tag{4.4}$$

$z = A^{-1}(-g - Nx)$

So a triangular fuzzy number solution is obtained $\tilde{x} = (x, y, z)$

5. Numerical Example

The fully fuzzy linear system is given as follow:

$(4,1,2)\otimes(x_1, y_1, z_1)\oplus(2,1,1)\otimes(x_2, y_2, z_2) = (-20,17,20)$

$(5,3,4)\otimes(x_1, y_1, z_1)\oplus(3,2,3)\otimes(x_2, y_2, z_2) = (-27,37,31)$

From the example problem above we can know that a matrix fuzzy $\tilde{A}$ is positive and vector $\tilde{b}$ is negative, so multiplication formula will be used on case 2, $\tilde{A}$ is posituve and $\tilde{x}$ is negative. This fuzzy linear system can be written in matrix:

$$\tilde{\boldsymbol{A}} = \begin{bmatrix} (4,1,2) & (2,1,1) \\ (5,3,4) & (3,2,3) \end{bmatrix} \quad \tilde{\boldsymbol{b}} = \begin{bmatrix} (-20,17,20) \\ (-27,37,31) \end{bmatrix} \quad \tilde{\boldsymbol{x}} = \begin{bmatrix} (x_1, y_1, z_1) \\ (x_2, y_2, z_2) \end{bmatrix}$$

fuzzy matrix $\tilde{A}$ and vektor $\tilde{b}$ is partitioned so thats is obtained:

$$A = \begin{bmatrix} 4 & 2 \\ 5 & 3 \end{bmatrix}, \quad M = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}, \quad N = \begin{bmatrix} 1 & 1 \\ 4 & 3 \end{bmatrix}$$

$$b = \begin{bmatrix} -20 \\ -27 \end{bmatrix}, \quad g = \begin{bmatrix} 17 \\ 37 \end{bmatrix}, \quad h = \begin{bmatrix} 20 \\ 31 \end{bmatrix}$$

Calculate Inverse of matrix A,

$$A^{-1} = \begin{bmatrix} 1.5 & -1 \\ -2.5 & 2 \end{bmatrix}$$

So that with equation (4.2), obtained

$x \quad = A^{-1}b$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1.5 & -1 \\ -2.5 & 2 \end{bmatrix} \begin{bmatrix} -20 \\ -27 \end{bmatrix}$$

$$= \begin{bmatrix} -3 \\ -4 \end{bmatrix}$$

$y \quad = A^{-1}(g + Nx)$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1.5 & -1 \\ -2.5 & 2 \end{bmatrix} \left(\begin{bmatrix} -20 \\ -27 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} -3 \\ -4 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$z \quad = A^{-1}(h + Mx)$$

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 1.5 & -1 \\ -2.5 & 2 \end{bmatrix} \left(\begin{bmatrix} 20 \\ 31 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} -3 \\ -4 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

So the solution from fully fuzzy linear system are $\tilde{x}_1 = (-3,2,1)$ and $\tilde{x}_2 = (-4,1,3)$.

6. Conclusion

From this article it can be concluded that the fully fuzzy linear system $\widetilde{\boldsymbol{A}} \otimes \widetilde{\boldsymbol{x}} = \widetilde{\boldsymbol{b}}$ is solved using invers matrix, based a new arithmetic triangular fuzzy number by determining the definition of matrix $\widetilde{\boldsymbol{A}}$, vector $\widetilde{\boldsymbol{b}}$ and vector $\widetilde{\boldsymbol{x}}$ is a positive or negative. Furthermore, it transforms fully fuzzy linear system $\widetilde{\boldsymbol{A}} \otimes \widetilde{\boldsymbol{x}} = \widetilde{\boldsymbol{b}}$ into $(\boldsymbol{A}, \boldsymbol{M}, \boldsymbol{N}) \otimes (x, y, z) = (b, g, h)$, then we can find out formula for applicable multiplication case operation by definition, so that the result obtained are compatible.

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