An Understanding of the Landscape of Behavioural Finance Biases and the Adoption of Mathematical Models in Strategy Formulation

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 Abstract

Persistent occurrences and phenomenon in the economic and financial systems about which there appears to be no sustainably effective solution are of concern to various stakeholders. Following the financial crisis of 2007-2008 there was a flurry of research activity geared towards explaining what the major causes of this and similar occurrences before were. Similar soul searching occurs after every such phenomenon, as evidenced when the South Sea Bubble burst centuries ago and other such crises that have occurred in the financial markets. This paper reviews some of the theories and what, in behavioural finance, are referred to as biases. An emerging strand of research is the field of neuroeconomics according to which medical imaging technology now allows us to look at brain activity as decisions are being made. The approach helps us to understand the nature and reasons for certain behavioural biases. Recent scientific studies have demonstrated that individuals with brain lesions that impaired emotional decision-making were more likely to behave as rational investors than individuals with normal brains. The paper reviews some of the quantitative or mathematical models used to explain behaviour underlying strategies used to make decisions in financial markets in particular and investment in general, namely, Prospect theory model, Quantitative Behavioural Model (QBM) and Igor's portfolio rule and investment strategies model.

Key words: Behaviour; Finance; Biases; Mathematical Models; Theories.

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1. Introduction

Use of mathematical and quantitative approaches as tools of explaining behaviour and phenomenon in finance has won admiration from academics and practitioners. Using   probabilistic theory to quantify risks and quadratic programming to derive solutions to economic and finance questions, Markowitz is among the earliest and most notable contributors to neoclassical finance and economics. He is among others, credited for developing the mean–variance portfolio selection model [1].

2. Methodology

 Literature review as a research method has been used in this study. As a method it is increasingly getting important, especially in social sciences partly because volatility and ever flexing environment and variables of study. The interrelatedness of the sub-sectors and areas of study make the approach relevant. In this particular study, for example, psychology, economics and finance, among other disciplines interface. In these various disciplines a lot of studies are being done and results published, thus the need to review current and past literature and integrate the findings for meaningful conclusions to be made. This presupposes more rigour on the part of the researcher than traditional literature reviews, which normally lack thoroughness.

3. Literature review

 About twenty years later Black and Scholes of the option pricing theory fame, offered an alternative by way of Itoˆ calculus and partial differential equations to explain issues of finance and economic behaviour [2]. Most specifically for behavioural finance, Daniel Kahneman, originally a psychologist deservedly won a Nobel prize in 2002 for his work on the prospect theory. However, evolving and application of mathematical and quantitative tools has been slow in behavioural finance, and this has been attributed to the fact that behavioural problems bring in highly unconventional and challenging features for which the known mathematical techniques have had very limited application , thus the need of new mathematical theories and approaches, instead of mere extensions of the existing ones [3] Behavioural finance may also be viewed from the perspective of being micro or macro. Micro behavioural finance analyses behavioural biases identifying individual investors considered as totally rational economic beings, homo economicus. It questions the authenticity of the   traditional rational decision-making theories. Its proponents argue that behavioural biases have a profound impact on decision-making and can make suboptimal decision- making and errors that directly contradict with traditional finance. Macro behavioural finance, on the other hand, analyses market anomalies that distinguish financial markets from efficient markets assumed by traditional finance and it questions the informational efficiency of markets. According to this approach to behavioural finance, financial markets are impacted upon by behavioural influences, such as market anomalies, bubbles, excess volatility and limited arbitrage [4].

3.1   *Traditional approaches*

The traditional approach to finance adopts a normative approach, outlining how the real world should function. However, it simplistically assumes players in the market to be rational Homo economicus, permanently seeking continuous dynamic optimization and equilibrium as postulated in accordance with the efficient market

hypothesis. It is the basis of Modern portfolio theory, mean variance analysis (expected returns, volatility of returns), Capital Asset Pricing Model and such other models of positive approach, which attempt to describe how the real world conceivably functions. Dissatisfied with some of the theories and models, some researchers adopted the behavioural finance approach. It is based on academic research in cognitive psychology and adopting the thinking that unlike the rational Homo economicus of the traditional finance school, players in the real world market are more likely to display irrational behaviour, guided more by emotions, optimism, pessimism, greed and fear than by rationality. A landmark development in explaining market behaviour in finance was the efficient market hypothesis (EMH) according to which in prices of securities or shares is impounded all available information at that point in time and that therefore such prices are optimal estimates of the true value of investment in these securities or shares [5]. The hypothesis posits that there are three levels of efficiency. The first is the weak form, which is when the decision maker only has past prices and returns as a guide. The second is the semi-strong form, when the decision maker has all information that is publicly available. The third and desirable form is the strong form, that is, when the decision maker has all public and private information as well.
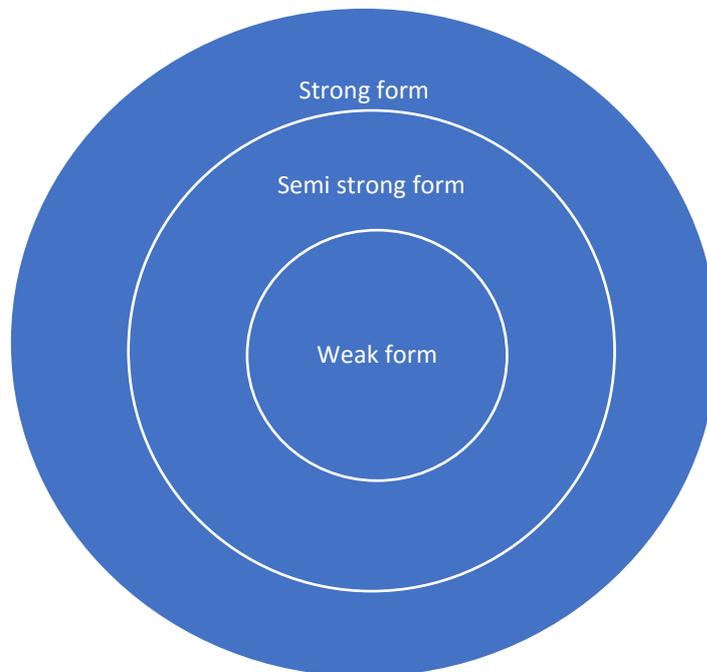
Figure I: Three Forms of Efficiency

Proponents of EMH premised their arguments on the key assumption that investors behave rationally on the basis of the information available and incorporated at the point in time, reinforcing this with the subsidiary arguments that there are incentives to gather and process information about securities and trade on the basis of the decision makers analysis until individual participant's valuation is similar to the observed market price and therefore that it can be concluded that when prices are thus determined, opportunities to earn above-normal rates of return on a consistent basis are limited. The EMH principal and subsidiary arguments as well are diluted and proven fallacious in light of the fact that on a daily basis and globally, there is evidence of irrational

behaviour in financial markets in particular and others in general. The traditional finance philosophy of rationality in decision making in financial markets is blurred by the inadvertent influence of beliefs, values and attitudes driven by socio-psychological and economic factors. These translate into behavioural biases which impact on the decisions eventually arrived at and this is so despite the multiplicity of financial models and technology available to facilitate decision making. Furthermore it is practically impossible to have all information that can enable one to precisely make valuation similar to the observed market price because like Donald Rumsfeld observed regarding ambigui*ty "Reports that say that something hasn't happened are always interesting to me, because as we know, there are known knowns; there are things we know we know. We also know there are known unknowns; that is to say we know there are some things we do not know. But there are also unknown unknowns — the ones we don't know we don't knowns." [6]*

3.2   *Multidisciplinary Approach to Behavioural Finance*

The role of law, politics, ecology as well as science and technology as factors influencing decision making by investors cannot be underestimated. Similarly, suspicions and uncertainties created when there is friction between the major global economic powers or those generated to join or leave an economic or political grouping greatly impact on investor behaviour and decision making. Regarding ecology, climatic change and the resultant effect on flora and fauna greatly impact on the human activities including agriculture, trade, manufacturing as well as the resultant decisions to invest or not, productivity and production of sectors and the returns of ventures invested in.
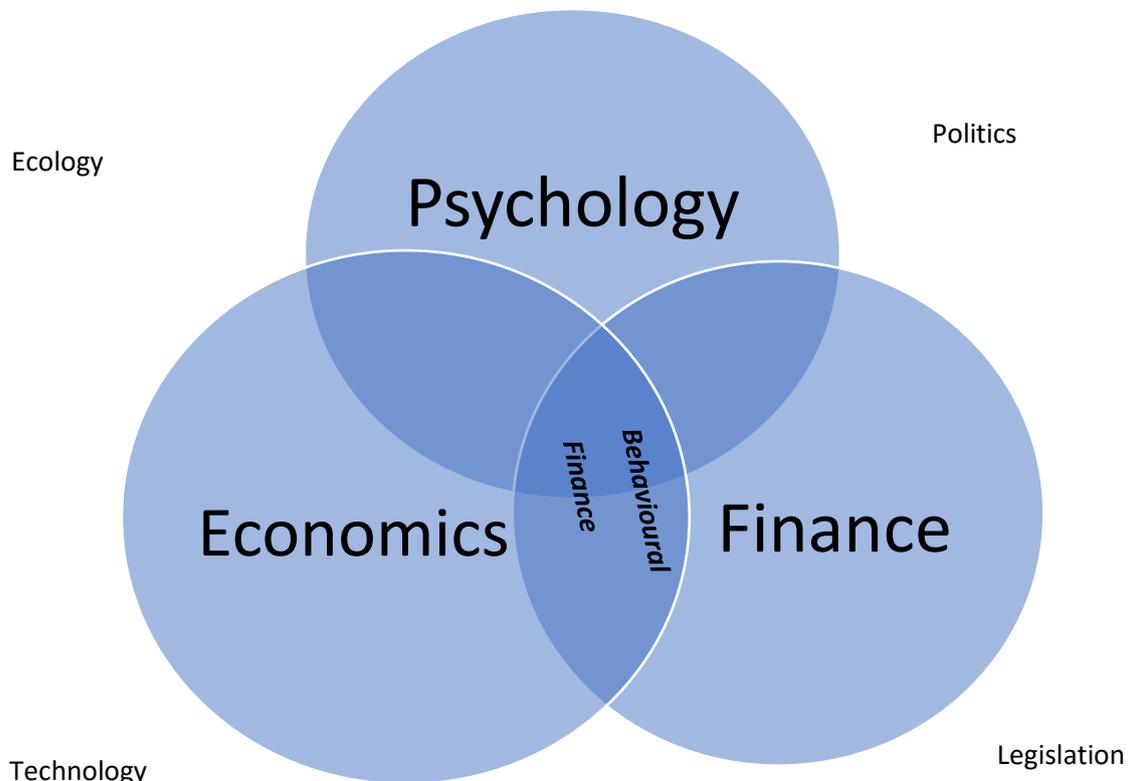

Figure 2: Multidisciplinary nature of Behavioural Finance

One emerging strand of research is the field of neuroeconomics. Medical imaging technology now allows us to look at brain activity as decisions are being made. This helps us to understand the nature of and reasons for certain behavioural biases. A recent study demonstrated that individuals with brain lesions that impaired emotional decision-making were more likely to behave as rational investors than individuals with normal brains. Other imaging studies have confirmed that the rational parts of our brain are in tension with the emotional or limbic sections of our brain [7].

4. Theoretical Framework of Biases Heuristics

As the behavioural finance discipline evolves various theories, and what have been referred to as biases have been used to explain the decision-making process and mindset of investors to arrive at what they believe may be winning strategies. One such approach has been to categorise as either general-purpose heuristics and special purpose heuristics. Examples of general-purpose ones are availability, causality, fluency, similarity and surprise. The six special purpose heuristics included attribution substitution, outrage, prototype, recognition, choosing by liking and choosing by default [8]. Another approach has been to divide biases them into four categories. The first category is that of those arising out of rigidity of opinion (cognitive dissonance) including conservatism, Status quo, confirmation, representativeness, illusion of control, disposition, belief perseverance and hindsight. The second category are those pertaining to information processing, namely, anchoring and adjustment, mental accounting, asset aggregation, framing, under-reaction and over-reaction, effect randomness, herding behaviour, and availability. The third category are emotional based biases including loss aversion, regret avoidance, prospect theory, over optimism, overconfidence, illusion of knowledge, self-attribution, self-control, loss aversion, status quo, endowment, regret aversion, escalation of commitment, gambler's fallacy and home bias. In the sections that follow below some of these biases are discussed.

4.1 Rationality

When a person has consistent preferences, he or she is said to be rational. However, it is difficult to find such a person in the real world because choices and pay-offs are logically premised on priors such as beliefs and predictably human behaviour is characterised by individuals appearing to have inconsistent preferences much of the time. Human behaviour is the potential and expressed capacity for physical, mental, and social activity during different phases of one's life cycles. Behavioural finance has its roots in the need to provide a discipline with a multidimensional approach to explaining behaviour of players in the market on the one hand and market phenomenon like depressions, bubbles and bursts, scams and crashes on the other. It has been defined as a study of human psychology and the rationality of making financial decisions not blurred by the traditional assumptions of expected utility maximization as had been assumed in efficient market hypothesis [9]. In comparison with mainstream approaches to finance decision making, which assume a fully optimal and efficient market, behavioural finance has, as a major objective, enabling a practical appreciation of financial decision-making and analysis as opposed to the former that assume that financial markets are inherently fully optimal and efficient. In recognition of its strategic and global importance of the behaviuoral finance discipline, the Behavioural Finance Working Group (BFWG) was established. It was from the brilliant works on cognition that Daniel Kahneman was awarded a Nobel Prize [10]. Many of the behavioural finance biases are generally

relative to the Bayes rule for updating our priors on the basis of new information which exists to influence the decision-making processes of players in the financial markets. The biases relate to how we process information to reach decisions and the preferences we have. As they are domiciled deep in our psyche it is pertinent that we appreciate not only their existence but also the potency with which they can impact on our decisions and eventually the results of our decisions. This way we may be able not only to reduce their influence and learn to work around them, but we may indeed be able to extrapolate the benefits or returns. Some of these are the subject of discussion below.

4.2 Prospect Theory

The likelihood of a certain outcome or outcomes resulting from an action or actions is a key driver or demotivator for a person acting in a certain direction or not. The prospects theory explains the apparent regularity in human behaviour when making decisions in situations of uncertainty. The theory states that individuals place much more weight on the outcomes that are perceived more certain than those that are considered merely probable. This aspect of the theory is referred to as the certainty effect. Accordingly, gambles are evaluated relative to a reference point, whereby decision makers analyse assumed gains and losses differently and that incremental value of a loss is larger than that of a gain, that is, the hurt of a $1000 loss is more painful than the benefit of a $1000 gain. The key conclusion of the prospect's theory is that misfortunes, losses and impediments have more prominent effect on inclinations than increases and preferences [11]. Psychology regarding decision making has been categorised as being either intuitive and deliberate mental processes. Intuitive processes are effortless, implicit and often emotionally charged. Deliberate ones are effortful, explicit and consciously controlled [12].
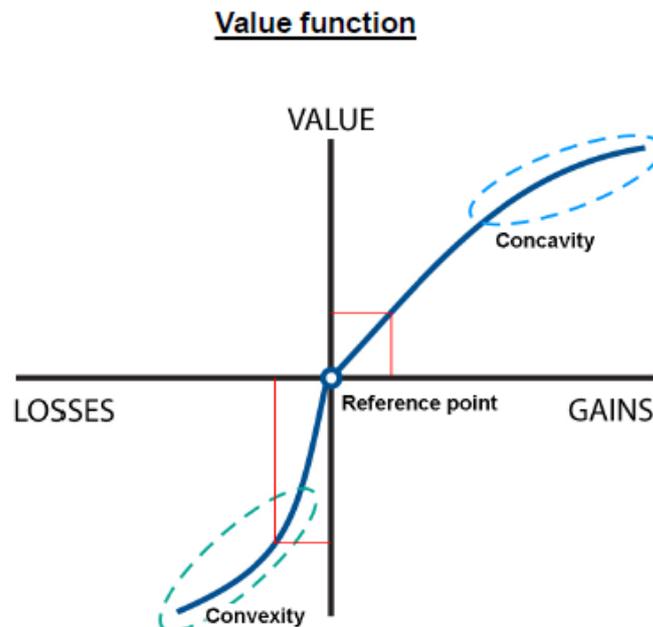

Figure 3: Value Function

Two main cornerstone features of the prospect theory are that value is assigned to gains and losses rather than to final assets (value function) and that probabilities are replaced by decision weight (weight function). Figures 1 and 2 below give a graphic description of these features. Along the number line continuum gains and losses originate from a reference point and the vertical line represents values. The value function is defined on deviations from a reference point and is normally concave for gains (implying risk aversion), commonly convex for losses (risk seeking) and is generally steeper for losses than for gains (loss aversion). The about 2 times steeper curve (2x) for losses compared to x for gains confirms the thinking in prospects theory that people feel losses(convex). much more than gains(concave). It is the relative values that are of significance when making decisions rather than absolute values. Regarding weighting the tendency is to over weigh probabilities of the higher and lower magnitude and under weigh those in the middle of the continuum. Like for the value function this makes the middle probabilities more realistic and therefore more reliable.

Weighting function
## (second cornerstone of prospect theory)
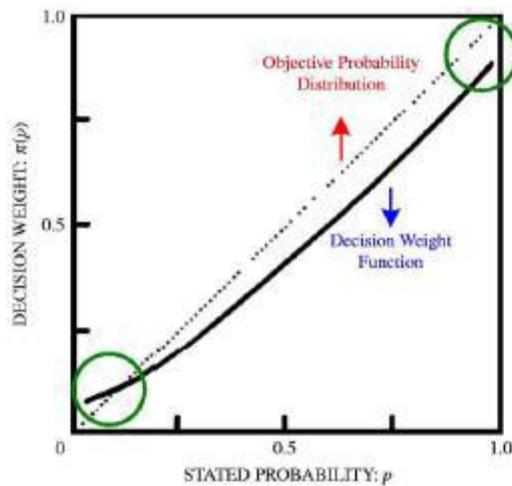
Figure 4: Weighting Function

The experiments carried out in respect of the prospect's theory confirms a distinctive fourfold pattern of risk attitudes, namely, risk aversion for gains of moderate to high probability and losses of low probability, and risk seeking for gains of low probability and losses of moderate to high probability. The function is normally concave for gains (implying risk aversion), commonly convex for losses (risk seeking) and is generally steeper for losses than for gains (loss aversion). The function predicts a distinctive fourfold pattern of risk attitudes: risk aversion for gains of moderate to high probability and losses of low probability, and risk seeking for gains of low probability and losses of moderate to high probability [13].

4.3 Cognitive Dissonance

The tendency to adjust beliefs to justify past actions is a psychological phenomenon termed as cognitive

dissonance. Research findings indicate that investors differ in their propensity to rely on a quick, affect-based decision mode, with the conscious planning based mode being a main feature of choosing among a repertoire of strategies available to them. With regard to investment decisions, individuals seek to reduce by adjusting their beliefs about the efficacy of past investment choices in the context of investment decision-making, especially when they find that their convictions and suspicions defy their expectations. They will accordingly tend to disregard new data that negates known convictions and choices [14].

4.4 Loss Aversion

It is predictable and logical for investors not to wish for loss and therefore to engage in mental processes geared towards eliminating or reducing the psychological impact of loss. They therefore have a natural desire to avoid admitting an error and incurring a loss. Such tendency is referred to as regret or loss aversion. It is the reason behind investors holding on to losing positions too long and staying out of a market that has recently generated losses even when gainful investment bargains exist. For example, an investor who buys shares for £1,000, which rises quickly to £1,500 in a market would be tempted to sell it in order to lock-in the profit, whereas if the investment dropped to £500, the investor would tend to hold it to avoid locking in the loss. The assertions above have been confirmed by earlier researchers who concluded that investors are about twice as averse to taking losses compared to enjoying an equal level of gains. Thus, most would require an even (50/50) chance of a gain of £2,500 in a gamble to offset an even chance of a loss of £1,000 for the proposition to be attractive [15]. The implication is that the general tendency is for investors with losing positions to prefer breaking even, that is investors tend to be highly risk-averse when facing a profit-making prospect which results in their selling to lock in the sure gain. The reverse is inevitably true, with investors being more risk tolerant or risk seeking when faced with incurring a loss, and they therefore tend to hold onto the investment in the hope that the market prices will favourably increase again. In some behavioural finance literature, the tendency for investors to sell a winning shares or stock and retaining a losing stock is called the *disposition effect*. It indicates that investors tend to sell winners and hold losers [16]. In a situation of what has been described as *myopic loss aversion*, too much importance is attached to routine daily fluctuations in the market, while it is sub optimal behaviour to abandon a long-term investment venture when faced with normal market behaviour. Accordingly, whereas risk aversion has for long been deemed to accurately describe investor behavior with gains. It is also true that when faced with a loss-making situation, investors often show risk seeking behavior.

4.5 Overconfidence

The tendency to believe that certain things are more likely than they really are is referred to as overconfidence. It is the complete opposite of many of the biases noted in behavioural finance research findings such as regret avoidance and loss aversion, among others. With that state of mind investors overstate their knowledge and competences, exaggerate their ability, under estimate and take risks in the hope that things will conform to their expectations [17]. At the extreme, either out of lack of knowledge or sheer recklessness, some investors do not diversify their investment. This stance is a result of an inflated view of their abilities. The quite common trait affects much of our lives with research indicating that most people rate themselves in the top third of the population. Indeed, many studies – of company CEOs, doctors, lawyers, students, and doctors' patients tend to

overrate the accuracy of their views of the future [18]. Despite some shortcomings, this attitude enables people to overcome or recover from life's disappointments far faster than they would have done if they did not have the trait. Quite often they overlook traditional financial theory which argues for holding diversified portfolios as a means of risk management, and with what appears 'misguided conviction', the overconfident investors go for what they believe are good or even excellent prospects. Many go for major contractual bids with quite sophisticated "theoreticians" whom they beat with intuition and displayed versatility! Overconfidence relates to how well individuals comprehend their own particular capacities and the points of confinement of their knowledge [19].

4.6 Self attribution bias

Some investors are so highly motivated by the skill and luck which they attribute to themselves ('self-attribution bias) that when faced with a positive outcome following a decision, they will view that outcome as a reflection of their ability and skill. However, when faced with the prospect of a negative outcome, it is attributed to bad luck or misfortune. Unfortunately, this attitude blurs the feedback process as it allows decision- makers to block out negative feedback and the resulting opportunity to improve future decisions. Many psychologists have concluded that a large portion of the financial specialists tend to dangerously overestimate their capacity to make correct decisions.

4.7 Herding behaviour

In life there is general belief that there is security or comfort in numbers. It is therefore not surprising that investors and analysts scan the environment to observe the actions and reactions of other players, and in response, head in the direction where the majority are moving, rationally or irrationally. This is likened to the movements of herds of animals, whether domestic or wild. The herding behaviour theory has been often used to interpret and predict price movements as well as to explain economic phenomenon like bubble build ups and bursts and other financial or economic crises. In effect herding could be a contradiction of the overconfidence bias and of information efficiency. It predictably leads to market stress by pushing asset prices away from their fair values as supported by the economic fundamentals, hence driving up market volatility [20]. The subprime crisis in the US is the most recent experience of herding behaviour with great ramifications on the global financial system. Between 2000and 2007, real estate prices in the country skyrocketed on account of investor's herding behaviour. They borrowed loans to buy houses for short-term profits. The temptation was not only the preserve of low-income households but also institutional investors who went in droves throughout the country to acquire mortgages. The relatively sophisticated institutional investors had the advantage of better access of information, which however was basically the same, and that was, that of historically low interest rate. Given the aversion bias in the context of stock market after bubbles, they homogeneously made decision to change their preference in housing investment. In their excitement banks heavily, and almost indiscriminately, lent out in the mortgage sector and their predicament was compounded by doing so on the basis of loose lending standards. Predictably this was followed by massive defaults, delinquencies and panic in the financial markets. Institutions ironically referred to as "too big to fail" had to be rescued through funding by government.

4.8 Groups and Crowds

It is important to appreciate how groups and crowds when, whether as natural persons or corporates are required to make decisions. Groups are typically appreciated on the basis of synergies which they are supposed to confer on the participants, including benefiting the competences or endowments of each participants. From the social point of view individuals prefer belongingness to isolation. In a 'ask the audience' feature of the Who Wants To Be a Millionaire quiz show research, the audience findings indicated that the range of knowledge and experience is diverse and that individuals give their opinion independently of the opinions of others [21]. According group-mind theory, individuals who became part of a crowd would lose all sense of self and responsibility when but they would gain a sentiment of invincible power due to their numbers [22].

4.9 Representativeness

It is logical to believe that a well-run company represents a good investment, which in turn implies that other factors follow accordingly. Therefore, representativeness is judgment based on stereotypes. The notion of 'representativeness bias reflects the case where decisions are made based on a situation's superficial characteristics (what it looks like) rather than a detailed evaluation of the reality. A common financial example is for investors to assume that shares in a high-profile, well-managed company will automatically be a good investment. This idea sounds reasonable, but ignores the possibility that the share price already reflects the quality of the company and thus future return prospects may be moderate. Another example would be assuming that the past performance of an investment is an indication of its future performance. In our psyche representativeness makes us believe that things that share similar qualities are quite alike. Players in financial markets especially technical analysts interchange a good company with a good investment and vice versa and when faced with firms having stocks characterised by a consistently good history of earning growth, they would tend to overvalue them. This is inherent in the various share or company valuation models such as dividend valuation and dividend growth models.

4.10 Framing

Unlike in traditional finance theory where it is recommended that all of an investor's investments should be treated as a single pool, or portfolio, such that risks of one or several cancel out with those of others through this diversification, in behavioural finance it is recommended that investors highly focus on the behaviour of individual investments , strategic business units or securities. Under this later approach the emphasised 'individualism' or 'narrow frames' heighten investor sensitivity to loss as opposed to 'aggregation' under which investors comfort themselves with the risk reducing covariance of the 'wide' frame, which enables investors to exhibit a greater tendency to accept short-term losses and their effects, with little consideration of the long term results.The main argument in favour of the behavioural finance approach is that there were justifications and merits for the investor to opt for a particular investment, strategic business units or securities and not others and have them as part of the portfolio. Record of the individual elements of the portfolio enable the investor to make more rational evaluation about whether individual securities or investments or market circumstances are changing so as to warrant change in the composition of the portfolio or strategy or both.

4.11 Hindsight Bias

The tendency to remember positive outcomes and repress negative outcomes is referred to as hindsight bias. There is therefore the temptation for investors to reminisce, with good memories, strategies that delighted them and not to dwell on the numerous times the strategy did not work. In statistical term an outcome's occurrence increases its perceived ex-ante probability of occurrence. The result is for investors to unambiguously violate rationality precepts in adjusting their estimates of the ex-ante probability of an outcome they know has occurred. This is if, but only if, they underestimate the possibility that the actual distribution of observed outcomes is a result simply of sampling error.

4.12 Mental Accounting

Mental accounting is a behavioural finance bias according to which there is a tendency for people to categorise their money into separate accounts based on a variety of subjective criteria such as the source of the money and the purpose for each account. It has also been referred to as the tendency to "put things in boxes" and track them individually. The mental accounting process involves three components. The first one involves establishing the intended outcomes of a decision or decisions and how subsequently these will be evaluated. Secondly the decision is made on as to what will be carried out in relation to specific accounts and how inflows and outflows relating to the activities relating to these accounts will be tracked. Finally, outcomes are periodically reviewed and balanced. In our mindsets we separate wealth and allocate it into various buckets or pools according to our objectives of having it and the risks related thereto, quite often according to time horizons. In practice individuals have criteria according to which they use or allocate their money, say on a monthly basis one may, using their discretion, save 30% for precautionary measures, 20% for investment and the remaining 50% for subsistence. For businesses they may be divided into strategic business units and funds allocated accordingly. In doing all these, whether for individuals or businesses there is criteria or policy adopted whether formally or informally. This way the individuals or businesses allocate their cash into distinctive mental accounts based on a mixture of subjective criteria, similar to the source of the cash and reason for every account.

4.13 Asset Segregation

The tendency to look at investment decisions individually rather than as part of a group is referred to as asset segregation and it makes an investor concerned more about those individual holdings which perform below expectation even when majority in the portfolio excel.

4.14 Anchoring

When making numerical prediction, estimates may be made by starting from an initial value referred to as an anchor to arrive at final judgements or answers, which may be suggested by the formulation of the problem, or it may be the result of a partial computation. In making investment decisions, as an introductory point, investors tend to remember the price they paid for a stock, and this information influences their subsequent decisions about what price they charge their customers. In forecasting when there are previous periods, there will be a closing figure for the most last period which can be adjusted to yield the final answer, although this may be

insufficient without well thought out assumptions [23]. In the financial sphere, values such as market index levels can act as anchors. Round numbers such as 5,000 points on the FTSE 100 Index, seem to attract disproportionate interest, despite them being numbers like any other.

4.15 Availability bias

This is a judgmental heuristic in which a person evaluates the frequency of classes or the probability of events by availability. It refers to the ease with which relevant instances come to mind.' Accordingly, recently observed events are believed to influence decision. A witness of a disastrous passenger plane crash in the recent past is more likely be very cautious when offered an opportunity to invest in the airlines sector or will be sceptical to travelling by plane than one who has not done so. In business investors who have observed a particular sector experiencing difficulty or an unstable financial market are likely to be sceptical before engaging in it. Psychologists refer to this as the 'availability bias'. It leads to overestimating the chances of disaster [24].

4.16 Conservatism bias

Even with new information available in the business decision making environment, some investors may be slow in responding and making changes to their original decisions or at most they may only slightly or partially adjust. This behaviour bias is referred to as conservatism bias. Thus, investors who buy shares in a high-profile company may be slow to adjust their view of the company's prospects even after the company's profitability deteriorates or if another company whose profile, they are not sure of presents an attractive proposition.

5. Mathematical Models in Behavioural Finance

Over time, behavioural finance has graduated through a predominantly narrative discipline, to one whose theories are augmented by use of quantitative or mathematical models. Those explored here are the Prospect theory model, Quantitative Behavioural Model (QBM) and Igor's portfolio rule and investment strategies model

5.1 Quantitative Behavioural Model (QBM)

Based on the assumption there is a market and after observing the behaviour of each of the participant in a market, a researcher will have knowledge that in that market there are many participants whose behaviour is very different from each other and that the actual market price is an aggregation of all the participants' expectations in the financial market. The proportion of each participant (called weight) determines how important they are in influencing the direction of the market.

The representative Quantitative Behavioural Model (QBM) model is:

$$E(Y_{QBM}) = W_1 * E(Y_{Investors}) + W_2 * E(Y_{Speculators}) + W_3 * E(Y_{Retail}). \ldots\ldots\ldots (i)$$

$E(Y_{QBM})$ is the actual market aggregation, $E(Y_{Investors})$ that of investors, $E(Y_{Speculators})$ that of the speculators and $E(Y_{Retail})$ that of the retailors.

Because of difficulty in finding the weights (W_1 W_2 and W_3), the optimisation algorithm called Swam Particle Swam Optimisation (PSO) model technique, (adopted after studying the behaviour of schools of fish and flocks of birds), which is a type of a stochastic optimisation technique, was used to find the weights.[25].

$$Vi(k+1) = w(k).Vi(k) + C1.R1(k).(Pbesti(k) - Xi(k)) + C2.R2(k).(Gbesti(k) - Xi(k)) \text{ (ii) } Xi(k+1) = Xi(k) +$$
$$Vi(k+1) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(iii)$$

Where $Vi(k+1)$ is the velocity of $(k+1)$ th iteration of *ith* individual, $Vi(k)$ is the velocity of kth iteration of *ith* individual, $w(k)$ is the inertial weight used as a trade-off between global and local exploration capabilities of the swarm. The objective function for the PSO algorithm is the root mean square error obtained from equation i. The parameters are W_1, W_2 and W_3. For the gold data, the proportion of long-term investors are about 52%, short term speculators are about 41% and stochastic retail investors are about 7 %, our weights predictions are consistent with the rarely available market data [26]. The PSO algorithm is based on three principles. Given a swarm of size n., each particle Pi (i =1, 2, . . . , n) from the swarm is characterized by: 1) its current position Xi (k)∈Rd , which refers to a candidate solution of the optimization problem at iteration k; 2) its velocity Vi (k)∈Rd ;and 3) the best position Pbesti (k)∈Rd that is identified during its past trajectory. Assume the best global position found over all trajectories that are travelled by the articles of the swarm to be Gbesti (k)∈Rd. Each of n particles fly through the d-dimensional search space Rd with a velocity V (k) i, which is dynamically adjusted according to its personal previous best solution Pbesti(k) and the previous global solution Gbesti(k) of the entire swarm. The velocity updates are calculated as a linear combination of position and velocity vectors. The particles interact and move according to the following equations:

5.2 Igor's portfolio rule and investment strategies model

In his contribution to mathematics as a tool of behavioural finance, Igor V. Evstigneev focused on developing a new theory of market dynamics and equilibrium, which he believed was a plausible alternative to the classical General Equilibrium theory (Walras, Arrow, Debreu, Radner and others). The characteristic feature of his work was to combine systematic application of behavioural approaches with the evolutionary modelling of financial markets. He believed that his approach would provide answers to the fundamental questions and problems pertaining to finance and financial economics, especially those related to equilibrium asset pricing and portfolio selection.He assumed A vector $\Lambda^i_o \in \Delta^k$ and a sequence of measurable functions with values in $\Delta K.\Lambda^i_t$ (S^t, H^{t-1}), t = 1, 2..., form an investment strategy (portfolio rule) Λ^i of investor i. For basic strategies, that is, those for which Λ^i_t depends only on S^t, and not on the market history Igor derived what he called portfolio rules basic thus: H^{t-1} = (P^{t-1},X^{t-1}, λ^{t-1}). To determine investor i's demand function, given a vector of investment proportions $\lambda^i_t = (\lambda^i_t$,1..., λ^i_t,K) of investor i, the i's demand function isX^i_t,k(P_t, X^i_{t-1}) =αλ^i_t, KB^i_t (Pt, X^i_{t-1}), where α is the investment rate. Short-run (temporary) equilibrium: for each t, aggregate demand for every asset is equal to supply: $\sum^n_{i=1}=$ X^i_t, k(P_t, X^i_{t-1}) = V_k, K = 1...,K. Igor's argued that portfolio rule is called a survival strategy if the investor using it survives with probability one (irrespective of what portfolio rules are used by the other investors! He further asserted that given a strategy profile (Λ^1,..., Λ^N), it can be said that the portfolio rule Λ^1 (or the investor 1 using it) survives with probability one if Inf $r^1 \geq 0$ (a.s.),(the market share of investor 1 is bounded away from zero a.s. by a strictly positive random variable).

6. Conclusion

Despite its short history compared to other disciplines like economics and traditional finance it has appealed to both intellectuals and practitioners by providing a foundation for evolving theories for a deeper understanding of the psychological processes involved in financial decision-making. No single discipline can explain behaviour in the markets and the interdisciplinary nature of behavioural finance would help in structured guidelines and rule of thumb investment choices for individuals by drawing attention to potential mental mistakes, hopefully leading to increased investment returns. Irrational behaviour is demonstrated not only in security markets but also in property, bullion, and commodities markets. No 'cure' may be arrived for all the biases, but awareness about them enables players to appreciate their existence and their effect, and help them possibly avoid the major pitfalls. A major component of finance is numeracy and it would be foolhardy for one to believe that when handling decision making about it one can avoid the application mathematical or quantitative models.

7. Recommendations

The emergent area of behavioural is critical to stakeholders in markets, whether financial or not. Prosperity of society largely relies on transactional exchanges and purchase as means of technological, social and economic transformation and appreciating the decision making process is crucial. The study has explored existing biases and a few, yet to be developed models. The increasingly globalised economic system require that more study is necessary for example the once closed economy of china has come to the forefront, to appreciate the impact of culture on market behaviour.

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