Mathematical Evaluation of Antiretroviral Therapy Effect in Ghana

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Abstract

Human Immunodeficiency Virus-Acquired Immune Deficiency Syndrome (HIV-AIDS) has continued to be a tremor to the health sector in Ghana. One person was first diagnosed with HIV-AIDS in the Eastern region part of Ghana in 1986. Subsequently, 41 people more were diagnosed HIV positive in the same year. The spread of this disease was tremendous to the extent that about 107, 333 and 2744 people were diagnosed by the end of the years 1987, 1988 and 1990 respectively; hence it was declared as epidemic in accordance with the status of World Health Organization (WHO). Because there is no cure for it yet, the Government of Ghana and other stakeholders have taken some interventions to reduce the spread of this epidemic. One of these interventions was the introduction of the antiretroviral therapy (ART) program in 2003 by the Ghana AIDS Commission (GAC) through the Ghana Health Service (GHS). This research was therefore purposed at the use of phase trajectories to evaluate the effect of the ART program on the HIV infections rate. The data on the number of HIV infected people per each year for the years 1986-2018 were collated from the reports given by Ghana AIDS Commission (GAC), WHO and UNAIDS published on their associate websites. The whole data set was divided into two, with data one and two illustrating the yearly number of HIV infected people for the periods before (1986-2002) and after (2003-2018) the introduction of the ART program respectively.

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Phase trajectory analysis was then performed on the various components of the two data sets. Hypothesis testing was finally performed on the means of the two data sets to confirm the results of the phase trajectory analysis. The various phase trajectories illustrating data one (1086-2002) indicated a tremendous yearly increase in the HIV infection rate from 1986 to 2002, while those of data two indicated a tremendous yearly decrease in the HIV infection rate from 2003 to 2018. It was further confirmed at 0.05 significant level that the average number of HIV infected people before the introduction of the ART program was greater than that after the introduction of the ART program. Hence, the ART program contributed much to the reduction of the HIV infection rate in Ghana. The nature of this study, being first in Ghana, and its findings will help in updating the Ghanaian government, citizenly and other stakeholders on the trend of HIV infection rate with the introduction of the ART program.

**Keywords:** Human Immunodeficiency Virus; Acquired Immune Deficiency Syndrome; Antiretroviral Therapy; HIV-Infected People; Phase Trajectories.

1. Introduction

1.1 What is Hiv/Aids?

Human Immunodeficiency Virus (HIV) is the virus that causes Acquired Immune Deficiency Syndrome (AIDS). The virus acts by weakening the immune system, making the body susceptible to other diseases and unable to recover from other diseases such as pneumonia, tuberculosis (TB), etc. A person can be infected with HIV for a long time without showing any symptom of the disease. Nonetheless, during that period before a person develops symptoms, he/she can transmit the infection through sex, delivery, breastfeeding, blood transfusions, sharing of injection needles. However, an individual is said to have developed AIDS when he/she presents with a combination of signs and symptoms after being tested HIV positive. This is as accounted by National AIDS/STI Control Program of the Disease Control Unit in reference [1].

1.2 Brief History of Hiv/Aids in Ghana

In March 1986, the first case of HIV/AIDS was reported in the Eastern region of Ghana. In January 1991, a more detailed report on HIV/AIDS in Ghana appeared in which 107 positive cases were recorded in 1987 and this increased to 333 by the end of March 1988. There was a tremendous increment of this to 2,744 by the end of April 1990, out of which 1,226 were said to have migrated to AIDS status. From the World Health Organization (WHO) annual report, the spread of HIV/AIDS was tremendous to the extent that Komfo Anokye Teaching Hospital alone was recording an average 50 cases monthly in 1991. Not all, following the introduction of an improved reporting mechanism, Ghana recorded 12500 AIDS cases by the end of 1994, placing Ghana second to Côte d’Ivoire in the West Africa sub region, where more than 16600 cases were recorded. Moreover, the HIV/AIDS was declared as epidemic in Ghana after the prevalence rate exceeded 1% in the general population, that is the standard set by WHO for an infectious disease to be generalized as epidemic. This is given in reference [2].

1.3 National Response
In 1985, the Government of Ghana established the National Technical Committee on the HIV/AIDS to advise it and implement measures to contain the epidemic. Following the confirmation of the first two cases of HIV/AIDS in 1986 and recognizing the potential impact that HIV/AIDS could have on the socio-economic development of the country, Government established the National AIDS Control Program (NACP) in 1987. The NACP was charged with the responsibility of coordinating the national response to the AIDS epidemic. Consequently, a short-term plan was developed for the prevention and control of HIV/AIDS which ran from 1987 to 1988. This was followed by the formulation of the first Medium Term Plan (MTP1) for HIV/AIDS prevention and control which effected from 1989 to 1993. A second Medium Term Plan (MTP2) was further developed to recognize the involvement of multilateral organizations, bilateral agencies and international and local NGOs so that the attention would not be focused on only health sector since the problem required a well coordinated multi-sectoral and multi-disciplinary response. Consequently, a joint team representing the Government, the United Nations Development Program (UNDP), the United States Agency for International Development (USAID) and other development partners recommended the establishment of a National Advisory/Coordinating Bodies to advise the Government on HIV/AIDS policies and other related issues. However, since the HIV/AIDS continued to be generalized as epidemic because the prevalence rate was still more than 1%, the cabinet responded by forming the Ghana AIDS Commission (GAC) in 2002 by Public Act 938 of Parliament to function as the supra ministerial and multi-sectoral body to advise and coordinate all HIV/AIDS related activities [3,4]. The GAC then coordinated through Ghana Health Service (GHS) and WHO for the introduction of the anti-retroviral therapy (ART) in 2003 [5]. The ART is a system of administering drugs to HIV-infected patients to attack the HIV at different stages of its life cycle to inhibit the virus replication, and thus bring the viral load down. According to WHO, standard anti-retroviral therapy (ART) consists of the combination of antiretroviral (ARV) drugs to maximally suppress the HIV virus and stop the progression of HIV disease [6]. In reference [7], the Hong Kong Integrated Treatment Centre carried a research on ART drugs adherence and its success, and it was found that the most effective aspect of ART is the Highly Active Anti-Retroviral Therapy (HAART) in that it is a very potent regimen which invariably inhibits viral replication to an undetectable level in the blood. In the research, it was also found that ART helps in the following ways:

- Restoration of the patient’s health and defense function.
- Declination of AIDS morbidity, hospitalization and mortality.
- Improvement of well-being.
- Improvement of the quality of life.
- Prolongation of AIDS patient’s survival.

Also, in reference [8], Palella, Delaney, Moorman and others investigated into the morbidity and mortality among patients with advanced HIV infection and found that the introduction of the ART gradually evolved the infection into a chronic and non-fatal condition. This was confirmed by Palmisano and Vella’s research on the success and challenges of ART through which it was detected that “with ART, HIV become a chronic illness in patients with continued treatment access”, as in reference [9]. In Ghana, before the introduction of ART, the generalized prevalence rate was trending up from below 1% in 1992 to 3.6% in 2003 and with regional variation
ranging from 0% (for the three Northern regions) to 10.6% (for Eastern region) in 1992 and from 1.3% (for Upper East region) to 5.3% (for Eastern region). However, when the ART was introduced in 2003, the national HIV prevalence rate started trending down from 3.6% in 2003 to 1.6% in 2018, while the regional prevalence rate dropped to less than 1% (for the three Northern regions) and to 2.9% (for Eastern region). This statistics is based on the HIV sentinel surveillance report in 2015, as in reference [10]. It is in this direction that this research is being conducted to investigate and evaluate the effect of ART on HIV/AIDS infection.

2. Materials and Methods

2.1 Data Collection

The data set for the study was retrieved from the annual reports on HIV status in Ghana published on different websites by the Ghana AIDS Commission (GAC) and other authorized agencies. The annual reports on the number of HIV infected people (I) and prevalence rate are based on HIV Sentinel Surveillance (HSS) report in Ghana. The estimated Ghanaian population (P) recorded in the corresponding years (2003 to 2018) was also retrieved from Worldometer, reference [11], which was one of the variables used to calculate the yearly number of susceptible people (S) out of the relation: S = Population - I. The whole data set on the yearly accumulated number of HIV infections from 1986 to 2018 was divided into two sets, with the first data set (table 1) entailing the data from 1986 to 2002, that is the period before the introduction of ART program. The second data set (table 2) also contains the data from 2003 to 2018, which is part of the period the ART program was introduced. The purpose for this division is to analyze and evaluate the HIV infection situation in Ghana before and after the introduction of the ART program. In other words, it is to determine the effect of ART on the HIV infection rate, that is whether it really contributed to a significant reduction of HIV infections or not.

2.2 Data Analysis

To get a fair idea about the HIV-AIDS situation in Ghana, descriptive analysis of the data was done through tabular and graphical illustrations of the various data sets. Graphical analysis, using Excel software, of phase trajectories was used to illustrate the various data in order to find out whether the introduction of ART was effective in the reduction of the HIV infection rate or not. This was followed by inferential analysis of the data whereby the significance of the role played by the ART on the HIV infection rate was further checked by using hypothesis testing based on t-distribution of unknown population variances with small sample sizes. This was to confirm results from the descriptive analysis. With this, tests on the difference between the respective population means (u1 and u2) of the first and second data sets with corresponding unknown population variances (σ1 and σ2) were carried out. Assuming the distributions of the two data sets are normal and that σ1 = σ2 = σ (the general population variance), the pooled t-test, often called the two sample t-test (Sp) was used.

The null hypothesis (H0) and the alternative hypothesis (H1) are given by:

\[ H_0: u_1 - u_2 \leq d_0 \quad \text{and} \quad H_1: u_1 - u_2 > d_0 \]

where \(d_0\) is a specific constant.
The test statistic \((T)\) was used for testing \(H_0\) because the respective sample sizes \(n_1\) and \(n_2\) of the first and second data sets are each less than 30. \(T\) has the t-distribution with \(n_1 + n_2 - 2\) degree of freedom (\(v\)) and is given by:

\[
T = \frac{(\bar{X}_2 - \bar{X}_1) - d_0}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}
\]

where \(\bar{X}_1\) and \(\bar{X}_2\) are the sample means of the first and second data sets respectively; and

\[
S_p = \sqrt{\frac{S^2_1(n_1 - 1) + S^2_2(n_2 - 1)}{n_1 + n_2 - 2}}
\]

where \(S^2_1\) and \(S^2_2\) are the sample variances of the first and second data sets respectively. With \(t\) denoting the calculated value of \(T\) and \(t_{a,v}\) denoting the tabular value of \(T\), \(H_0\) is rejected in favor of \(H_1\) when \(t > t_{a,v}\), otherwise \(H_0\) is accepted at significance level of \(5\% = 0.05\). If \(H_0\) is rejected, it indicates that there would be enough evidence to conclude that the mean number of HIV infected people for the period 1986 – 2002 is significantly greater than the mean number of HIV infected people for the period 2003 – 2018. If \(H_0\) is accepted, it indicates that there would be enough evidence to conclude that the mean number of HIV infected people for the period 1986 – 2002 is significantly less than or equal to that for the period 2003 – 2018 [12].

3. Results

3.1 Descriptive Analysis of the Data

In the case of descriptive analysis, the various data sets were analyzed using tables and graphs (phase trajectories). The whole data set was divided into two with data set one representing the yearly accumulated number of HIV-infected people from 1986 to 2002 and data set two representing the yearly accumulated number of HIV-infected people from 2003 to 2018.

3.1.1 Tabular illustration of the First Data set

The data of the yearly accumulated number of HIV-infected people from 1986 to 2002, that is the period before the introduction of ART program, is as entailed in table 1 below. The first column is headed by the year; the second column, headed by time (\(t\)) in years, illustrates the number of years elapsed since the first HIV incidence in Ghana; the third column represents the estimated Ghanaian population (\(P\)) per year; the fourth column represents the year number of HIV infected people (\(I\)) and the fifth column also represents the yearly number of susceptible people (\(S\)) determined by finding the difference between the corresponding population and \(I\).
Table 1: Data on yearly HIV infections before the introduction of the ART program.

<table>
<thead>
<tr>
<th>Year</th>
<th>Time (t) in years</th>
<th>Population (P)</th>
<th>I</th>
<th>S = Population – I</th>
</tr>
</thead>
<tbody>
<tr>
<td>1986</td>
<td>1</td>
<td>13104616</td>
<td>42</td>
<td>13104574</td>
</tr>
<tr>
<td>1987</td>
<td>2</td>
<td>13480997</td>
<td>107</td>
<td>13480890</td>
</tr>
<tr>
<td>1988</td>
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<td>13853171</td>
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<td>13852838</td>
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<td>1994</td>
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<td>16337298</td>
<td>12500</td>
<td>16324798</td>
</tr>
<tr>
<td>1995</td>
<td>10</td>
<td>16760926</td>
<td>217892</td>
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<td>2001</td>
<td>16</td>
<td>19293392</td>
<td>578802</td>
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<td>19786307</td>
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<td>19113573</td>
</tr>
</tbody>
</table>

3.1.2 Excel Graphical Analysis of the First Data Set

All the phase trajectories were plotted using the Excel software.

I against S

First, plotting the number of HIV infected people (I) against the number of susceptible people (S = Population – I) at any moment in time using table 1, the Excel output is as shown figure 1 above. The nature of the deep back line illustrates the trend of the number of HIV infected people right from 1986 to 2002.

Looking at the shape of the phase trajectory (curve) of the graph in figure 1, the line of the trajectory generally keeps on moving up from left to right. However, the steepness of the trajectory is not uniform as it is less steep in an up-and-down manner for S – values from 13.1 to about 16.3 million but the steepness is very high for S – values between 16.3 – 16.5 million. The steepness reduces to almost horizontal level for S-values from 16.5 to about 18.2 million and then increases sharply for S-values from 18.2 million onwards. This means that the number of HIV infected people (I) was generally increasing with increasing susceptible people, only that the infection rate was very low at instances where the trajectory is less steep and tremendous at instances where the trajectory is very steep.
Figure 1: Graph of number of HIV infected people against number of susceptible peoples from 1986 to 2002.

$I$ and $S$ against time ($t$)

Figure 2: Graph of number of HIV- infected people / susceptible people against time ($t$) in years

To effectively compare HIV infection rate to the susceptible rate as the years went by, $I$ and $S$ are plotted on the vertical axis against time ($t$) on the horizontal axis, as shown in figure 2 below. The green-colored line illustrates the trend of the number of susceptible people while the blue-colored line illustrates the trend of the number of
HIV infected people from 1986 to 2002.

From the graph, the direction of the line representing the number of susceptible people (S) is in an upward trend from left to right as the years went by. The same thing applies to the line for the number of HIV infected people (I), only that the steepness of the susceptible line is steadier while that of the infected line is not uniform. The steepness of the infected line is in slightly up-and-down manner for the years 1986 to 1994 and then increases sharply from there to 1995. The steepness reduces almost to the horizontal level from 1995 to 1999 and then increases sharply again from there on wards. In short, it is clear in table 1 that the number of HIV-infected people tremendously increased right from 1986 to 2002, and this is confirmed by the ascending nature of the respective trajectories in figures 1 and 2 above.

3.1.3 Tabular illustration of the Second Data set

Table 2 below generally illustrates data set of the number of HIV infected people (I) and those removed from the infected people recorded for the period 2003-2018. The Ghanaian population (P) per year for the period 2003-2018, as retrieved from reference [11], is as shown the third column. It is then followed by the yearly number of HIV infected people (I) recorded in the respective years. The fifth column is headed by the number of susceptible people calculated by using the relation \( S = P - I \). The sixth column is headed by the number of HIV infected people who have been enrolled onto the ART program and as such removed (R) from the susceptible group. The last column shows the yearly HIV prevalence rate, that is how quick people are infected with HIV every year, as reported by HSS.

<table>
<thead>
<tr>
<th>Year</th>
<th>Time (in years)</th>
<th>Population (P)</th>
<th>I</th>
<th>( S = P - I )</th>
<th>R</th>
<th>HIV Prevalence Rate (%)</th>
</tr>
</thead>
<tbody>
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<td>2003</td>
<td>1</td>
<td>20301686</td>
<td>736839</td>
<td>19564847</td>
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<td>20189613</td>
<td>2028</td>
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<td>577369</td>
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<td>702328</td>
<td>21245451</td>
<td>7338</td>
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<td>585667</td>
<td>21939992</td>
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<td>221941</td>
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<td>25879749</td>
<td>75044</td>
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<td>2015</td>
<td>13</td>
<td>27849205</td>
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<td>27597308</td>
<td>69616</td>
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<td>16</td>
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<td>334713</td>
<td>29547171</td>
<td>113133</td>
<td>1.6</td>
</tr>
</tbody>
</table>

3.1.4 Excel Graphical Analysis of the Second Data Set
Plotting $I$ against $S$ using the values in table 2, the trajectory (curve) obtained is as shown in figure 3 below. The vertical axis is scaled with the number of HIV infected people while the horizontal axis is scaled with the number of susceptible people. The nature of deep black line described by the plot determines the trend of the HIV infection following the introduction of the ART as the susceptible people increased in number year after year.

**Figure 3:** Graph of number of HIV infected people ($I$) against number of susceptible peoples from 2003 to 2018.

From figure 3, from left to right, the trajectory descends steeply during the first three instances, ascends sharply for a moment and then descends steeply again until it almost got to the horizontal axis before it exhibited up-and-down trend between the sixth and eleventh dots along the horizontal axis. From there, the line started ascending gently to the end. This means that the number of HIV infected people ($I$) averagely decreased as the number of susceptible people ($S$) increased, only that at an instance, it went up slightly from about 580000 to about 700000 before descending tremendously to about 240000, then maintained an up-and-down trend between 220000 and 270000 and then started increasing slightly while the number of susceptible people kept on increasing drastically.
\textbf{I against R}

Plotting $I$ against the number of people removed ($R$) from the HIV infected group as a result of being enrolled onto the ART program, the Excel output is as in figure 4 below. The blue-colored line shows the trend of HIV infection relative to that of the people who are enrolled onto the ART program.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.png}
\caption{Graph of number of HIV infected people ($I$) against number of people removed($R$) from I.}
\end{figure}

From the graph, figure 4, the line started descending steeply, ascended steeply from the third dot to the fourth dot, then descended sharply to the sixth dot after which it exhibited an up-and-down trend till the tenth dot where it dropped back to the eleventh dot and then started ascending gently through the dots to the end. This means that the number of HIV infected people ($I$) was reducing as the number of people enrolled on the ART ($R$) increased during the first three sections of the ART program and then increased slightly. After that, $I$ reduced drastically again before it started exhibiting up-and-down variations as $R$ continued to increase. However, $I$ and $R$ together reduced (that’s where the line dropped back) before $I$ began increasing slightly as $R$ increased tremendously.

\textbf{$I$ and $R$ against $t$}

Now plotting $I$ and $R$ on the vertical axis against the Year/ Time ($t$), on the horizontal axis, figure 5 below is the
Excel output obtained. The blue colored line illustrates the trend of HIV infection as the years (time) went by while the red colored line illustrates the trend of at which HIV infected people are enrolled onto the ART program.

![Graph of number of HIV infected people (I) / people removed (R) from I by being enrolled onto the ART program against time (t) in the years from 2003 to 2018.]

**Figure 5:** Graph of number of HIV infected people (I) / people removed (R) from I by being enrolled onto the ART program against time (t) in the years from 2003 to 2018.

In figure 5 above, the shape of the trajectory for I indicates reduction of HIV infections from about 740000 in 2003 (first year of ART introduction) to about 580000 in 2005 (year three) after which it went up again to about 700000 in 2006 (year four). It then reduced drastically over a two-year period (through 2007 to 2008) and then exhibited an up-and-down variations over a six-year period (from 2009 to 2014) where it started increasing in the subsequent years. However, the number of people enrolled onto the ART program (R) kept on increasing since its inception in 2003 right from 197 through the years to about 73000 in 2012 and then reduced slightly to about 67000 in 2013, from which it started increasing year after year. This implies that the HIV infection rate averagely keeps on reducing since the introduction of the ART program in 2003.

**I and S against t**

Plotting I and S against Year/ Time (t), the Excel output are the graphs shown in figure 6 below. The black line is the trajectory for the number of susceptible (S), and its nature illustrates the trend of susceptible people while the blue line and its nature illustrate the trajectory for the number HIV infected people (I) and its trend respectively since the introduction of the ART program. Each value of I is multiplied by $10^{1}$. 

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In figure 6, the trajectory for \( I \) decreases in the same manner as in figure 5, while the trajectory for \( S \) ascends from left to right from 2003 (year one) to 2018 (year 18). This also shows that the rate of HIV infection decreased as against an increasing susceptible rate from 2003 onwards.

**HIV Prevalence (%) against Year/Time (t)**

Plotting HIV Prevalence (%) on the vertical axis against Year/Time (t) on the horizontal axis using Excel software, figure 7 below is obtained. The nature of the trajectory illustrates the trend of HIV prevalence rate (%) in Ghana after the introduction of the ART program.
Figure 7: The graph of yearly HIV prevalence rate (%) from 2003 to 2018.

In figure 7 above, the line illustrating the HIV prevalence rate exhibits an up-and-down trend as it descends from left to right as the years went by. This means that the HIV prevalence rate generally kept on going down even though it is up-and-down in certain moments. Generally, the number of HIV infected people \( I \) is clearly in descending order right from 2003 onwards, as portrayed in table 2, hence a positive ART effect on the HIV infection rate. This could be an indication that the ART helps in the reduction of the HIV infection rate, and this is in line with GAC accession, as in reference [10]. With these exhibitions, it can happen that, as long as the ART program is highly patronized, there could be a time \( t \) that there would be no new HIV infections, that is the moment that the trajectories illustrating number of HIV infected people would cut the horizontal axis in figures 3 and 4. This is in agreement with Palmisano and Vella findings, as in reference [9]. The positive effect of the ART program is also confirmed by the descending nature of the graphs in figures 3 and 4.

3.2 Inferential Analysis of Data

In order to confirm and generalized the results from the descriptive analysis, the inferential analysis is carried out. Using hypothesis testing based on t-distribution of unknown population variances with small sample sizes, let \( u_1 \) and \( u_2 \) be the population means of the first and second data sets with corresponding unknown population variances \( \sigma_1 \) and \( \sigma_2 \). Hence, using significance level of \( \alpha = 0.05 \), the null hypothesis \( (H_0) \) and the alternative hypothesis \( (H_1) \) are stated as follows:

\[ H_0: u_1 - u_2 \leq d_0 \quad \text{and} \quad H_1: u_1 - u_2 > d_0 \]
where \(d_0\) is a specific constant = 0

\[H_0: \mu_1 \leq \mu_2 \quad \text{and} \quad H_1: \mu_1 > \mu_2\]

Assuming that the distributions of the two data sets are normal and that \(\sigma_1 = \sigma_2 = \sigma\) (the general population variance), hence using the sample t-test \((S_p)\), the test statistic \(t\) of \(T\) is given by:

\[
t = \frac{\bar{X}_2 - \bar{X}_1 - d_0}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}
\]

Where:

\[
\bar{X}_1 = \frac{\sum X_1}{n_1} = \frac{42 + \ldots + 672734}{17} = \frac{2877724}{17} = 169277.8824
\]

\[
\bar{X}_2 = \frac{\sum X_2}{n_2} = \frac{736839 + \ldots + 334713}{16} = \frac{2877724}{16} = 384171.625
\]

\[
S_p = \sqrt{\frac{n_1(n_1-1)+n_2(n_2-1)}{n_1+n_2-2}} = \frac{\sqrt{4.680259563\times10^{10}(17-1)+3.645091894\times10^{10}(16-1)}}{17+16-2} = 204435.1237
\]

\[=> t = \frac{(384171.625-169277.8824)-0}{204435.1237 \sqrt{\frac{1}{17} + \frac{1}{16}}} = 3.0178
\]

Now from the \(t\)-distribution table and with degree of freedom \((v) = n_1 + n_2 - 2 = 17 + 16 - 2 = 31\), the \(T\) statistic from the table is \(t_{0.025,31} = 2.042\). Since \(t = 3.0178 > t_{0.025,31} = 2.042\), \(H_0\) is rejected in favor of \(H_1\). Hence, at 0.05 level of significance, there is enough statistical evidence that the mean of data set one is significantly greater than that of data set two. Since \(H_0\) was rejected in favor of \(H_1\) at 0.05 level of significance, there is enough statistical evidence that the mean of data set one is significantly greater than that of data set two. Hence, there is enough statistical evidence that the average HIV prevalence rate for the period 1986-2002 is greater than the average HIV prevalence rate for the period 2003-2018. And since the ART program was introduced in 2003 by the GAC and other stakeholders in Ghana, and was still being used through the period 2003-2018, there is enough statistical evidence to conclude that the ART program really contributed to the reduction of the HIV infection rate in Ghana.

4. Conclusions

The number of the HIV infected people was on a tremendous increase right from 42 in 1986 to 736839 in 2003 with respective national prevalence rate of below 1% to 3.6%, before the introduction of the ART program in 2003. But after the introduction of the ART program, the number of the HIV infected people started reducing from 736839 in 2003 to 334713 in 2018 with respective national prevalence rate of 3.6% to 1.6%. Even though the reduction trend of the HIV infection rate for the period 2003-2018 was in the up-and-down manner, this is
still an indication that the introduction of the ART has helped in the drastic reduction of the HIV infection rate. From the graphical analysis, the ascendency (from left to right) of the plots for the number of HIV infected people in the phase trajectories illustrating the data before the introduction of the ART program shows that the number of HIV infected people kept on increasing year after year before the introduction of the ART program; while the descending nature (from left to right) of the plots for the number of HIV infected people in the phase trajectories illustrating the data after the introduction of the ART program shows that the number of HIV infected people was in a reducing manner after the introduction of the ART program. This is also an indication that ART program has massively contributed to the reduction of HIV infection rate in Ghana. The inferential analysis performed on the two data sets resulted in the rejection of the null hypothesis ($H_0$) at significance level of 0.05, there is enough statistical evidence that the average HIV prevalence rate for the period 1986-2002 is greater than the average HIV prevalence rate for the period 2003-2018. This implies that the HIV infection rate before the introduction of the ART program was higher than the HIV infection rate after the introduction of the ART program. Hence, the confirmation that the ART program really contributed massively to the reduction of the HIV infection rate in Ghana. We therefore recommend the Ghanaian Government and other stakeholders should organize educational programs on the need to patronize the ART program. Also, the public should be psyched against stigmatization of HIV/AIDS victims so that people will feel free to declare their HIV/AIDS status and thereby helping the health specialist to easily identify them and offer the necessary assistance. Last but not the least, there were a lot of difficulties in retrieving the data from the various sources, in that there were differences in the data recorded.

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