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Multi-parameter Model for Computing Head Loss Development in Granular Media Deep-Bed (Depth) Filters Used for Production of Potable Water

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Abstract

This research work focuses on the effects of clogging due to accumulation of solids in the pores of packed beds used as granular media filters for the production of potable water. Modelling of the filtration process will require an assessment of the changes that take place within the filter media pores during filtration including its effects on filtrate quality and on the development of head loss during filtration. In gravity (deep-bed) filtration the driving force is a measure of the pressure drop for overcoming media resistance. Filter averaged data obtained from pilot filter test runs was used for the formulation of a logarithmic model relating head loss rise [HLR] to filter run length [FRL]. A multi-parameter model was subsequently conceived and formulated by coupling this logarithmic model with Carmen-Kozeny's clean bed head loss equation. The model is made up of seven key parameters namely, head loss, filter run length, friction factor, media depth, filtration velocity, bed porosity and filter grain size. The model was subsequently verified and validated. It would be applied in the computation of head loss development in gravity filters at increasing run times during a filtration cycle. This innovative time-varying model will serve as a predictive tool for design and operation of dept filters. It will therefore facilitate the work of designers and managers of water treatment works.

Keywords: multi-parameter; clogging; packed beds; modeling; head loss; gravity filtration.

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1. Introduction

In filter operation, straining is regarded as one of the principal solids removal mechanism. Suspended solids concentration is the dominant influent raw water characteristic and is monitored principally through turbidity and head loss measurements. Head loss rise (HLR) in a filter is a function of the amount of material (particles) accumulated in the interstices and pore space of the filter. The larger the quantity of particulates removed from the separation process, the higher the head loss rise.

In deep-bed granular filters, the water quality, media depth, media grain size distribution (GSD), filtration rate (velocity), flow rate, head loss including the deposition (accumulation) of suspended solids in the pores of filter beds, vary with time and location.

A filtration model for the purification of water for potable and industrial use must have both spatial and temporal characteristics to clearly define the movement and time components respectively

Specifically, the objective of this research is to conceive and formulate a time-variable model that could be used for computing head loss development during a cycle of filter run in granular media deep-bed filters.

The model that would be developed in this research will be of the fixed-frame structured system.

2. Conceptual Framework

2.1 Background

Granular media deep-bed filtration can be conceived as proceeding in three stages, namely initial, ripening and ultimate. In the initial stage, the incoming fines (particulates) impinge on the clean media grains and are collected (deposited) through clean collector-fines interaction to form a monolayer which cover the collectors. The ripening stage results in the formation of multilayer deposits on the monolayer through fine-fine interactions. The ultimate stage is finally reached when the incoming particulates are obstructed or blocked by the multiplayer. This results in the formation of cake like deposits and a termination of filter run as the pores are filled up by accumulated particles.

The rate of flow through a filter is directly proportional to the driving force and inversely proportional to the filter media resistance and the solids retained by the media. This can be expressed as:

$$\text{Flow rate (Q)} \propto \text{driving force / media resistance} \quad \ldots\ldots\ldots\ldots\ldots \text{(2.2a)}$$

In gravity filtration, the driving force is a measure of the water head or pressure drop for overcoming filter resistance. The pressure drop is measured as the difference in water level above the filter media and in the clear well to which the effluent discharges. Flow through a packed bed can be analyzed by classic hydraulic theory. Bernoulli's energy equation for flow through a conduit (2.2b) indicates that the energy is balanced between two points along any section and takes into account the head loss. Darcy Weisbach's equation (2.3) predicts head loss for flow in a conduit.

$$P_1/\rho g + v_1^2/2g + z_1 = P_2/\rho g + v_2^2/2g + z_2 + h_L \quad \text{. . . . .} \quad (2.2b)$$

$$h_L = fL\, v^2/2gd \quad \text{. . . . .} \quad (2.3)$$

where, P_1, P_2 = pressure, kg/m^3

v_1, v_2 = velocity, m/s

z_1, z_2 = height of water above a datum level, m

ρ = density of water, kg/m^3

g = gravity constant (9.81), m/s^2

h_L = head loss, m

f = Darcy friction factor

L = pipe length, m

v = flow velocity, m/s

2.2 Clean Bed Head Loss Equations

The author in [1] observed that this equation (2.3) could be applied for flow of water through a clean bed when it is assumed that the pores between the sand grains form capillaries. For the capillary model, during filtration when clogging occurs, the pore space and specific surface area of the capillaries will decrease with time while the number and the length of the pores will remain the same.

The author in [2] modified the Darcy-Weisbach equation for head loss in a conduit to take into account the porous nature of a granular filter bed of uniform size.

The resulting equation (2.4) known as Carmen-Kozeny equation is a linear relationship between resistance and filtration rate. It is strictly true when the eddying resistance is negligible compared to the viscous forces [3].

$$h_C = f\, x\, (1 - e)v_s^2/e^3 g d_p \quad \text{. . . . .} \quad (2.4)$$

h_C = clean bed head loss = friction loss through bed of particles of uniform size d_p, (m)

x = depth of filter media, m

e = porosity of bed

v_s = filtering velocity, the velocity of the water just above the bed, (m/s)

g = gravitational acceleration, (m^3/s^2)

d_p = diameter of filter media grains, (m)

The remaining term f is a friction factor related to the coefficient of drag around the particles. In the usual range of filter velocities (laminar flow), this can be calculated from the formula.

$$f = 150(1-e)/Re + 1.75 \quad \ldots\ldots\ldots\ldots\ldots \quad (2.5)$$

where R = Reynolds Number = $\phi \rho_w v_s d_p/\mu$

ϕ = Shape factor

ρ_w = density of water, (kg/m^3)

μ = dynamic viscosity, (kg/m.s)

The author in [4] showed that author in [5] used dimensional analysis to develop a similar equation (2.6) for a clean bed:

$$h/x = 1.067 C_d\, v^2/gd\psi e^4 \quad \ldots\ldots\ldots\ldots\ldots \quad (2.6)$$

where h = head loss in bed of depth x with face velocity v.

e = bed porosity

d = characteristic diameter of bed particles

ψ = particle shape factor

C_d = Newton's drag coefficient = $24/Re + 3/R^{0.5} + 0.34$

x = bed depth

Carmen-Kozeny's model and Rose's (equation) are applicable for flow through clean filter beds. When a packed bed is used for removal of suspended matter, the porosity of the bed changes with time due to accumulation of solids in the voids.

When solids begin to accumulate during filtration through a clean porous filter medium, the porosity of the bed decreases resulting in an increase in head loss. The rate of solids accumulation, and therefore the rate of head loss change is a function of the nature of the suspension, the characteristics of the media, and filter operation methods [6].

The authors in [7] carried out a study on particle clogging in porous media. An empirical single-parameter model and a two-parameter model were derived for predicting the increased clogging that is always observed in the top segment of deep-bed filters. Published data from six (6) filtration studies were analyzed and applied in the coupling of flow hydrodynamics with permeability reduction during clogging through an empirical representation of deposit morphology.

The present research will concentrate on the formulation and development of a hydrodynamic models that will integrate the most important processes that are crucial to depth filtration of water through porous granular media.

2.3 Limitations and constraints

The limitations and constraints of the study included the problem of sourcing enough funds for procuring additional equipments and materials for the research. However data obtained from experiments carried out using available pilot filters and operational filtration plants facilitated the timely completion of the study.

3. Materials and Methods

3.1 Materials

Filter media used were silica sand and pumice (porous volcanic rock). These are bulk filtration media, which are porous materials with pores used as particulate collectors. Grain size range (d) for silica sand (specific gravity 2.65, uniformity coefficient (uc) 1.5) is 0.5 to 1.7 mm. In most public water supplies, granular beds of silica sand are used. It is readily available in Nigeria, relatively cheap and satisfactorily purifies water.

Pumice used (specific gravity 1.6, uniformity coefficient 1.4 similar to values recommended for bituminous and anthracite coal) has a grain size of 1.0/1.1 mm. It is used as an upper layer. Pumice is available in Eritrea, Italy and Turkey in commercial quantities and has been proved suitable for dual-media and multi-media filtration [8]. Anthracite and bituminous coal are available in substantial quantities in the numerous coal measures of Nigeria. They can be used in place of pumice as upper layers on top of sand for dual-media filters. The choice of media types for the investigation was based on current practices in the water treatment industry. The raw waters used were of low turbidity (range 2.85 to 10.0 NTU). In these investigations, pilot filters designed to operate identically with full-scale existing filters were used [8,9,1,10].

3.2 Methodology

3.2.1 Empirical Modeling

Empirical modelling is a process in which analytical representations are formulated using discrete data acquired from experiments. The choice of functional form depends on use to be made of the formula [11]. Filter averaged data extracted from optimised head loss rise versus filter run length (time) curves were analyzed to ascertain the best functional form of empirical model that fits it analytically. The data were appraised for conformance with four model forms namely constant, linear, logarithmic and exponential [6].

Regression analysis was carried out to obtain empirical relationships for filtration parameters being modelled. The parameters are: head loss rise (h_r) and filter run length (t).

The equation for non-linear regression is:

$$E[Y/X] = A + B\,g(x) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.1)$$

Where E is the mean value function for the generalized regression equation of Y on X; g(x) may be algebraic, exponential, logarithmic or any other function of x. A and B are regression coefficients. The correlation coefficient R was used as a measure of the accuracy of the linear prediction [12].

3.2.2 Head Loss Rise as a Function of Filter Run Length

Figures 3.1, 3.2, and 3.3 show relationships between head loss rise and filter run length. Maximum HLR and FLR values are in accordance with values proposed by previous researchers [4,1]. The normalized curves show similar patterns indicating that a relationship for describing the rate of change of head loss rise with filter run length exists.

3.2.3 Two-Parameter Head Loss Rise Model

The log-log graphical plot for HLR and FRL displayed in Figures 3.4 (Table A) and 3.5 (Table B) were straight lines indicating that the desired relationship is logarithmic of the form:

$$x = \beta\, y^{\gamma} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.2)$$

$$\text{Or} \qquad \text{Log}\, x = \log \beta + \gamma \log y \ldots\ldots\ldots \quad (3.3)$$

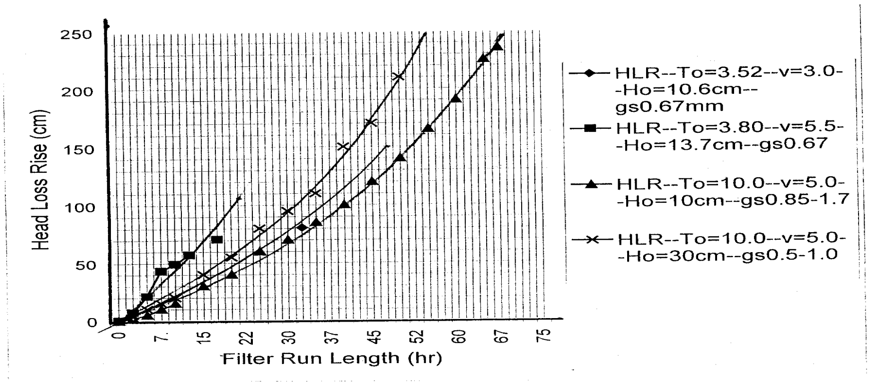

Figure 3.1: Head Loss Rise as a Function of Filter Run Length for Mono-Media Filters (Filtration Rate 3.0 – 5.5 m/hr)

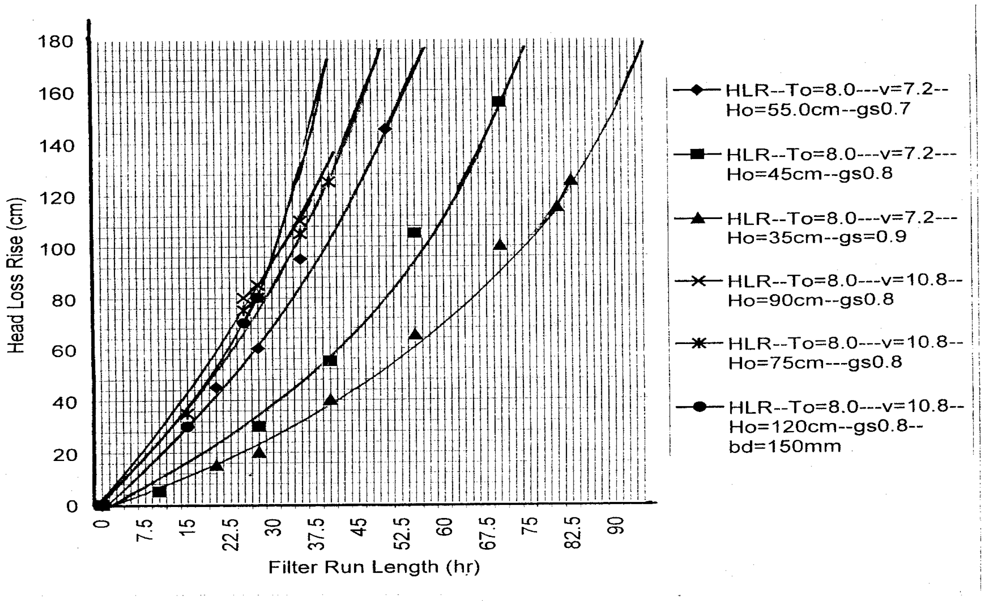

Figure 3.2: Head Loss Rise as a Function of Filter Run Length for Mono-Media (sand) Filters (Filtration Rate 7.2 – 10.8 m/hr)

The relationship for mono-media and dual-media filters as indicated in figures 3.4 and 3.5 is of the form:

$\text{Log } h_r = \log \beta + \gamma \log t$ . . . . (3.4)

Or $h_r = \beta t^{\gamma}$ . . . .(3.5)

where h_r = head loss rise in filter (cm)

t = filter run length (hr)

β, obtained from the intercept, and γ the slope of the log-log plot are constants.

Equation 3.5 is for a filtration process that operates under a constant filtration rate and shows that the relationship between head loss rise h_r and filter run time t is logarithmic. The specific resistance, which results from clogging of the filter is the exponent γ, while β is a function of the medium resistance. γ is a coefficient which depends on the density of the suspension, the suspended solids concentrations in the influent raw water, and the filtration velocity, while β is dependent on the characteristics of the filter media and influent raw water.

For mono-media (Fig 3.4), the model is:

$\text{Log } h_r = 0.375 + 1.01 \log t$ . . . . (3.6)

Or $h_r = 2.371\, t^{1.01}$ . . . . (3.7) where $\log \beta = 0.375$ or $\beta = 2.371$ (cm), and $\gamma = 1.01$

Similarly the model for dual-media filters is:

$$\text{Log } h_r = -0.56 + 1.152 \log t \quad \ldots\ldots\ldots\ldots \quad (3.8)$$

$$\text{Or} \quad h_r = 0.275\, t^{1.152} \quad \ldots\ldots\ldots\ldots \quad (3.9)$$

Where $\log \beta = -0.56$ or $\beta = 0.275$ (cm), and $\gamma = 1.152$

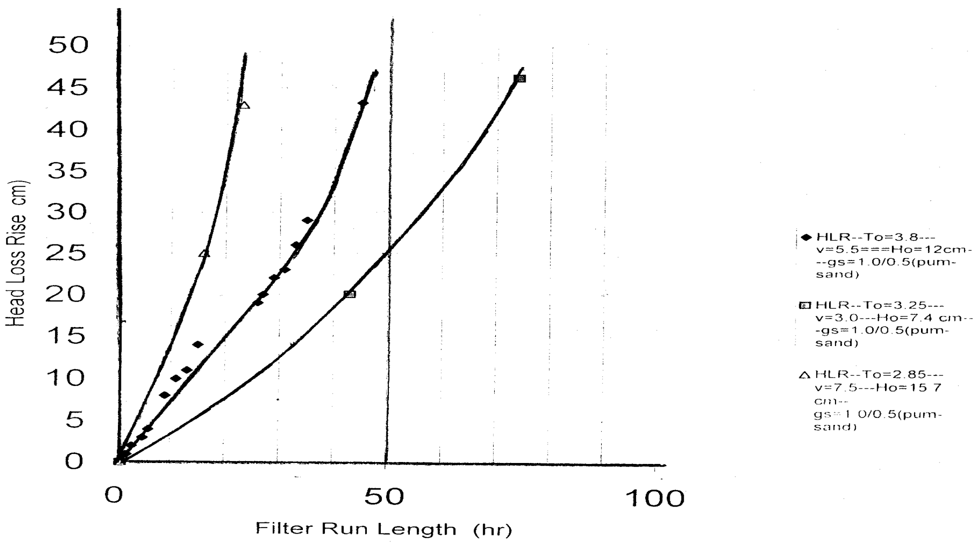

Figure 3.3: Head Loss Rise as a Function of Filter Run Length for Dual-Media (pumice-sand) Filters (Filtration Rate 3.0 – 15.0 m/hr)

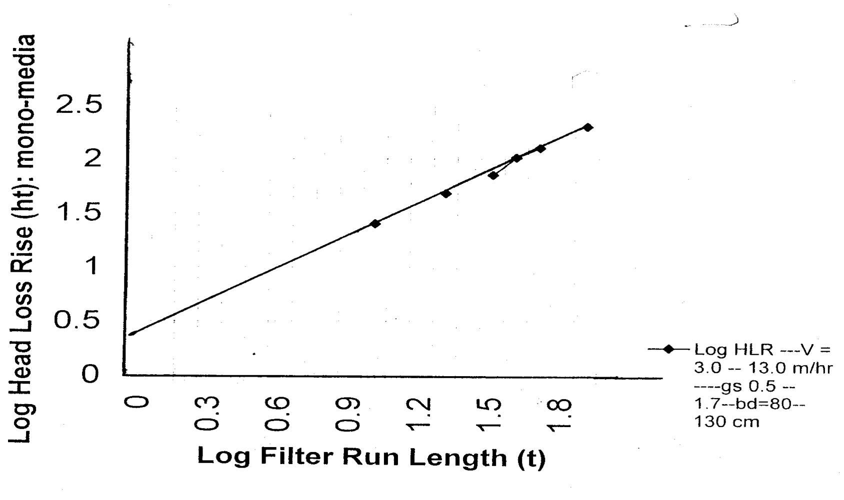

Figure 3.4: Log Head Loss Rise as a Function of Log Filter Run Length for Mono-Media (sand)

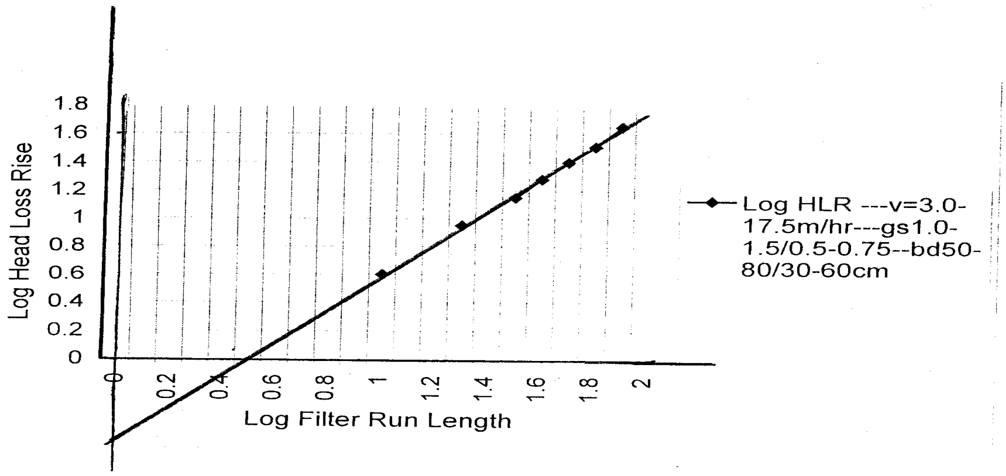

Figure 3.5: Log Head Loss Rise as a Function of Log Filter Run Length for Dual-Media (pumice/sand or anthracite/sand) Filters

3.2.4 Multi-Parameter Head Loss Development Model

A multi-parameter model was formulated by coupling the logarithmic head loss rise model equation 3.5 with Carmen-Kozeny's clean bed head loss equation. The resulting model has both spatial and temporal components. It allows for the determination of head loss for varying media depths (x) and filter run time (t).

This model can be written in word form as:

Head loss at time t (h_t) = Head loss rise (h_r) + Clean bed head loss (h_c) . . . . (3.10)

OR $h_t = h_r + h_c$ . . . . (3.11)

ie, $h_t = \beta t^\gamma + f x (1 - e)v_s^2/e^3 g d_p$ . . . . (3.12)

For mono-media (sand) the equation is:

$$h_t = 2.371 t^\gamma + f x (1 - e)v_s^2/e^3 g d_p \quad \ldots\ldots \quad (3.13)$$

while for dual-media (pumice-sand and anthracite-sand) it is:

$$h_t = 0.275 t^\gamma + f x (1 - e)v_s^2/e^3 g d_p \quad \ldots\ldots \quad (3.14)$$

4. Verification and Validation of the Formulated Filtration Models

Verification of the formulated multi-parameter model was carried out using data from literature. Carmen-Kozeny equation was applied using the relevant data given in Table 4.1 to obtain a clean bed head loss (CBHL) of **27 cm** for the mono-media (sand) filter and **14.5 cm** for the dual-media (pumice/sand) filter. These values agree substantially with the measured values **(25 +/- 3.5 cm** and **12 +/- 3.2 cm** respectively) shown in Table 4.1.

Table 4.1: Pilot Filter Process Data [8].

Parameter	Mono-media (sand)	Dual-media (pumice/sand)
Influent turbidity (NTU)	10.0	4.61 +/- 1.6
Filter run length (hour)	50	44
Water production per cycle (WPPC) (m^3/m^2)	250	242
Filtering velocity (m/hour)	5.0	5.5
Grain Size (mm)	0.5 –1.0	1.0 / 0.5
Bed Depth (cm)	70	60 / 30
Clean bed head loss (cm)	25 +/- 5.0	12 +/- 3.2
Shape factor **	0.85	0.75 / 0.85
Specific gravity	2.65	1.6 / 2.65
Porosity **	0.4	0.45

** Assumed average values from literature [1].

The head loss through the filters at increasing time steps during the filtration cycles were computed using the formulated multi-parameter model equations 3.13 and 3.14. The results are shown in Tables 4.2 and 4.3.

5. Results and Discussion

The results indicate that terminal head loss values are **150.3 cm and 36 cm** for the indicated filter run lengths for mono and dual-media filters respectively. This conforms to the indicated standard requirement that terminal head loss in granular media deep-bed filters may not exceed **240 cm**. reference [4] allowed for a maximum head loss of **250 cm**.

The validity of the formulated multi-parameter model is assured from the results of the verification carried out above. The computed values are within the ranges of measured data; hence the models are considered appropriate for the prediction and evaluation of head loss development in alternative granular media deep-bed filter configurations. The approach most commonly used for calculating head loss in a clogged filter has been to compute it with a modified form of the equations used for computing clean water head loss. This method requires that media porosity must be estimated for various degrees of clogging which is complex and prone to a wide margin of error.

Table 4.2: Head Loss Development in Mono-Media (sand) Filter

Time (t) (hour)	Head loss rise (h_r) (cm)	Clean bed head loss (h_c) (cm)	Head loss at time t (h_t) (cm)
0	0	27	27
5	12	27	39
10	24.3	27	51.3
15	36.5	27	63.5
20	48.9	27	75.9
25	61.2	27	88.2
30	73.6	27	100.6
35	86	27	113
40	98.4	27	125.4
45	110.8	27	137.8
50	123.3	27	150.3

Table 4.3: Head Loss Development in Dual-Media (pumice/sand or anthracite/sand) Filter

Time (t) (hour)	Head loss Rise (h_r) (cm)	Clean bed head loss (h_c) (cm)	Head loss at time t (h_t) (cm)
0	0	14.5	14.5
5	1.8	14.5	16.3
10	3.9	14.5	18.4
15	6.23	14.5	20.7
20	8.7	14.5	23.2
25	11.2	14.5	25.7
30	13.8	14.5	28.3
35	16.5	14.5	31
40	19.3	14.5	33.8
44	21.5	14.5	36

This research is significant considering the fact that the head loss of a filter is a key criteria for determining when a filter run should be terminated. At the maximum head loss the flow reaches the design minimum (turbidity break through stage). The filter at this stage must be shut down and backwashed prior to the commencement of a new filtration cycle. Terminal head loss can now be accurately determined using the multi-parameter model formulated in this study.

6. Conclusion and Recommendation

This research resulted in the formulation of a multi-parameter model by coupling the formulated two-parameter logarithmic model with Carmen-Kozeny clean bed head loss equation. The model has temporal and spatial components that allow for assessment of head loss at each step increase in time and depth.

It is recommended that the models formulated in this research should be applied in the design of new granular media deep-bed filters and also for design of upgrading works for existing filters that perform below original design capacity.

Application of this innovative time-varying multi-parameter model will provide a solution to the problem of determining head loss development in granular-media deep-bed (depth) filters.

The models would also serve as management tools for predicting and diagnosing filter performance in water treatment practice.

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