



A Critique of the Hungarian Method of Solving Assignment Problem to the Alternate Method of Assignment Problem by Mansi

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Abstract

Matching highly skilled people to available position/job is a high-stake task that requires careful consideration by experienced resource managers. The assignment problem is a special type of the transportation problem, it is an important subject discussed in real world. In this research, two methods were used to solve the assignment problem (i.e Hungarian method, and the Alternate method of assignment) and the results were compared. The result from these comparison shows that both method gives the same optimal solution but the Alternate method yielded the optimal solution in few steps which saves time. The data collected was also analyzed and solved using statistical software "TORA 2.0" which gave similar result of 881(in millions of Naira).

Key words: Optimal solution; Assignment problem; Feasible solution; Alternate method; Hungarian method.

1. Introduction

The assignment problem arises because available resources (such as men, machines etc.) have varying degree of efficiency for performing different activity.

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Therefore, cost, profit or time of performing different activity is different. Thus, the problem is how the assignments should be made so as to optimize the given objective. The assignment problem is completely specified by its two components, the assignment which represents the underlying combinatorial structure and the objective function to be optimized which models “the best possible way.”

The assignment problem is a particular type of the transportation problem where the objective is to assign a number of resources to an equal number of activities so as to as to minimize total profit of allocation. In the assignment model, worker s represent sources and jobs destination. The supply (demand) amount at each source (destination) exactly equal 1. The cost of “transporting” worker i to job j is c_{ij} . In effect, the assignment model can be solved directly as a regular transportation model (or as a regular linear programming problem). Nevertheless, the fact that all the supply and demand amounts equals 1 has led to the development of a simple solution method called Hungarian method. Although, the new solution method appears totally unrelated to the transportation model, the algorithm is actually rooted in the simplex method, just as the transportation model is.

There are several methods of solving the assignment problem such as

- i. Enumeration method
- ii. Transportation model
- iii. Hungarian method
- iv. The new Alternate method of [7]

Here, we shall discuss briefly on only the two methods of interest; these methods are:

i. Hungarian method

This method was developed by [5], a Hungarian mathematician. In order to use this method, one needs to know only the cost of making all the possible assignments. Each assignment problem has a matrix (table) associated with it. It may be noted that the assignment problem is a variation of the transportation problem with two characteristics

- a. The cost matrix is a square matrix
- b. The optimum solution for the problem would be such that there would be such that there would be only one assignment in a row or column of the cost matrix.

This method works on the principle of reducing the given cost matrix to a matrix of opportunity cost. Opportunity cost here shows the relative penalties associated with assigning resource to an activity as opposed to making the best or least assignment.

ii. The Alternate method of assignment

This method was developed by [7] as an alternate method to the Hungarian method. In this method, the minimum cost entry in each row of the cost matrix is used for feasible assignment in cases where there is no ties

and each row has a unique minimum cost entry. In cases where there is no unique minimum cost entry for each row, the difference in minimum cost entry and next minimum cost entry for that row is used for assignment (i.e. the row with the maximum cost difference).

However, the Hungarian method is an easy method of finding the feasible (optimal solution) to an assignment problem according to researchers. But for the purpose of this research, we will restrict ourselves to the recent developed method by [7] called the Alternate method of assignment.

1.1 Statement of the problem

Given n resources (or facilities) and n activities (or jobs) an effectiveness (in terms of cost, profit, time etc.) of each resource (facilities) for each activity (job), the problem lies in assigning each resource to one and only one activity (job) so that the given measure of effectiveness is optimized.

Let x_{ij} denote the assignment of facility i to job j such that

$$x_{ij} = \begin{cases} 1, & \text{if facility } i \text{ is assigned to job } j \\ 0, & \text{otherwise} \end{cases}$$

Then, the mathematical model of the assignment problem can be stated as follows.

The objective function is to

$$\text{Minimize } z = \sum_{i=1}^n \sum_{j=1}^n C_{ij} x_{ij}$$

Subject to the constraints

$$\sum_{i=1}^n x_{ij} = 1 \quad \text{for all } i \text{ (resource availability)}$$

$$\sum_{i=1}^n x_{ij} = 1 \quad \text{for all } j \text{ (activity requirement)}$$

Where $x_{ij} = 0$ or 1 and

C_{ij} represents the cost of assignment of resource i to activity j .

1.2 Significance of the study

The purpose of this research is to show the effectiveness of the assignment model in the University of Port Harcourt. In some establishment it appears that taking decision is on the basis of mere trail. That is why the

topic was chosen and an institution (University of Port Harcourt) requires assignment model application.

1.3 Scope of the study

The scope of the study is to focus on how jobs are assigned to different companies so as to minimize the total cost of assignment.

1.4 Limitation of the study

The study is limited to the tenders submitted for project assignment in the University of Port Harcourt.

ASSUMPTIONS

- i. All the companies bidding for the jobs are competent enough to deliver satisfactorily.
- ii. All the companies bidding for the jobs are registered companies.

1.5 Literature review

An efficient method of finding optimal solution to an assignment problem without making a direct comparison to every solution was developed by [5]. This was named the Hungarian method. This method was capable of reducing the cost matrix such that at least one zero in each row and column will be obtained; thus optimal assignment will be made possible where opportunity cost is zero. However [6] came up with a computational approach that makes use of duality in an effective manner. He also stated that there exist optimality in an assignment problem if and only if it is complete after every possible transfer. Reference [10] researched on assignment of workers to jobs in an economy with co-ordination frictions. This was done by assigning heterogeneous workers to heterogeneous jobs. He stated that the model exhibits equilibrium and must be extended to a dynamic framework if considered in quantitatively series. Reference [8] provided a novel solution with the application of constraint programming and artificial intelligence to bridge the gap between the need for high quality matches and the need for timeless. Reference [7] In his thesis on transportation problem developed a new alternate method of solving assignment problem optimally. He developed an algorithm for it and also proved with numerical example that his method yields similar result with that of Hungarian but with fewer steps which makes it easier. The work by [11] modeled driver`s characteristics with respect to their choice of routes by the use of a new fuzzy assignment problem. [4], worked on the geometry of simplex method and applied it to the assignment problem. In their research, the Hungarian algorithm and the Gale-shapley algorithm was implemented into MATLAB where preference matrix from an excel file was taken in as input and an optimal and stable result was obtained. Reference [13] worked on the extension of the parallel auction algorithm that was proposed by [1]. Their work was focused on the case where the availability of local information and how it always converges to an assignment problem and the benefit is being maximized within a linear approximation of the optimal one. Reference [9] considered a quadratic assignment problem to model local problems in logistics by taking into consideration the cost of placing new items in certain places and the interaction between all items. He also stated that the Heuristic method, branch and bound method and complete enumeration method can be used in solving quadratic assignment where $n \geq 25$. Reference [3] applied Hungarian algorithm to baseball team

selection. They achieved this by assigning the most appropriate player to their respective position on the field in order to obtain an optimal team.

2. Methodology

This section provides discussions on the proposed method for solving the assignment problem.

Assignment problem is a special type of the transportation problem which is also resource allocation problem. Here, we have n jobs to be performed with n companies and the problem is how to optimally assign these jobs to different companies involved. Depending on the merit or potential of the company, who will be able to accomplish the task in different time. Then, the objective function is assigning the different jobs to different companies is to find the optimal assignment that will minimize the total time taken to finish all jobs by the companies. The reader should see [7] for a detailed algorithm for both maximization and minimization problem. Also the reader should see [12] for the Hungarian method algorithm of solving the problem.

2.1 Data collection, analysis and conclusion

In this section, we shall consider a computational study and comparison of the new alternate method of assignment by [7] and the Hungarian method for solving University of Port Harcourt tender-job assignment problem.

2.2 Data collection and analysis

Six projects were to be undertaken in University of Port Harcourt. These projects were put on tender and six companies tendered in their bids. After introspection and evaluation by institution physical planning department, it was realized that each of these seven companies who placed their bids on these projects can execute any of the seven projects in the institution.

The six projects are faculty of education building (ED), Engineering block (EN), Pharmacy building (PH), Management building (MA), Central lecture hall (CE) and Health science building (HS). The seven companies are as follows: Sam & Vic Ltd (SA), Jipdant Association Ltd (JA), Minno Engineering Ltd (ME), VICTENIK Ltd (VL), Hytect Global Ltd (HG), Kafenal Ltd (KA) and Fine Job Ltd (FJ).

The bids (in millions of Naira) of tenders to the various project is given in Table 2.1

The problem now is how the projects should be assign to the companies to minimize the total cost of undertaking the projects.

Now the cost matrix is not balanced, hence we add one dummy column (OL) with zero cost entries in the column. The cost matrix so obtained is given in Table 2.2

Table 2.1: Bids (in millions of Naira) of tenders to various projects.

Companies	Projects					
	ED	EN	PH	MA	CE	HS
SA	126	207	254	245	214	243
JA	229	238	242	228	213	285
ME	118	253	306	218	245	216
VL	172	247	218	248	217	243
HG	309	207	105	136	194	139
KA	99	168	220	140	215	116
FJ	95	174	168	145	249	98

Table 2.2: Table of cost matrix

Companies	Projects						
	ED	EN	PH	MA	CE	HS	OL
SA	126	207	254	245	214	243	0
JA	229	238	242	228	213	285	0
ME	118	253	306	218	245	216	0
VL	172	247	218	248	217	243	0
HG	309	207	105	136	194	139	0
KA	99	168	220	140	215	116	0
FJ	95	174	168	145	249	98	0

2.3 Solution to the Assignment problem using Alternate Assignment Algorithm

Now we select row SA and select that column for which row SA has minimum cost. Row SA has minimum value in column ED. So we write company SA under column 1 and project ED under column 2. In a similar way, we select all rows and find their minimum cost entry for the respective columns. This is shown below

Here, company JA and HG has unique cost entries as it doesn't occur again and hence we assign project CE and PH to company JA and HG respectively. Next, delete column 3 and 5. Again select the minimum cost entries for the remaining projects as shown below

Table 2.3.1: Selection Procedure

Column 1	Column 2
SA	ED
JA	CE
ME	ED
VL	ED
HG	PH
KA	ED
FJ	ED

Table 2.3.2: Minimum cost entries

Companies	Projects				
	ED	EN	MA	HS	OL
SA	126	207	245	243	0
ME	118	253	218	216	0
VL	172	247	248	243	0
KA	99	168	140	116	0
FJ	95	174	145	98	0

Table 2.3.3: Selection procedure

Column 1	Column 2
SA	ED
ME	ED
VL	ED
KA	ED
FJ	ED

Since, company SA, ME, VL, KA and FJ has a single minimum cost ED, we now take the minimum cost difference for of them.

Here, minimum cost SA is $(207-126) = 81$, cost difference for ME is $(216-118) = 98$, that of VL is $(243-172) =$

71, that of KA is $(116-99) = 17$ and that of FJ is $(98-95) = 3$. Since ME has the maximum difference, we assign project ED to company ME and we delete column ED and row ME. Next, we select the minimum cost entries for the remaining projects as shown in Table 2.3.4 below.

Table 2.3.4: Minimum cost entries

Companies	Projects			
	EN	MA	HS	OL
SA	207	245	243	0
VL	247	248	243	0
KA	168	140	116	0
FJ	174	145	98	0

Table 2.3.5: selection procedure

Column 1	Column 2
SA	EN
VL	HS
KA	HS
FJ	HS

Here, company SA has unique cost entry as it doesn't appear again and hence we assign project EN to company SA. Next, we see that company VL, KA and FJ has same minimum cost entry. Hence we take the minimum cost difference. Minimum cost difference for VL is $(247-243) = 4$, minimum cost difference for KA is $(146-116) = 24$ and that of FJ is $(145-98) = 47$. Since FJ has the maximum differences we assign project HS to company FJ and delete the row and column respectively as shown in Table 2.3.6 below.

Table 2.3.6: Minimum cost entries

Companies	Projects	
	MA	OL
VL	248	0
KA	140	0

In this step, only one column (except dummy column) remains along with two rows VL and KA. So we assign KA to MA (since it has the minimum value). Next, only VL is left which will be assigned to dummy OL. Finally each company is assigned a unique project which is shown below.

Table 2.3.7: Final selection & costs

Column 1	Column 2	Cost
SA	EN	207
JA	CE	213
ME	ED	118
VL	OL	0
HG	PH	105
KA	MA	140
FJ	HS	98
TOTAL		881

2.4 Solution to the Assignment Problem using Hungarian Method

Table 2.4.1: Minimum cost entries

Companies	Projects						
	ED	EN	PH	MA	CE	HS	OL
SA	126	207	254	245	214	243	0
JA	229	238	242	228	213	285	0
ME	118	253	306	218	245	216	0
VL	172	247	218	248	217	243	0
HG	309	207	105	136	194	139	0
KA	99	168	220	140	215	116	0
FJ	95	174	168	145	249	98	0

A walk through the Hungarian algorithm gives the following computational values.

Table 2.4.2: Minimum cost entries

Companies	Projects						
	ED	EN	PH	MA	CE	HS	OL
SA	31	39	149	109	20	145	0
JA	134	70	137	92	19	187	0
ME	23	85	201	82	51	118	0
VL	77	79	113	112	23	145	0
HG	214	39	0	0	0	41	0
KA	4	0	115	4	21	18	0
FJ	0	6	63	9	55	0	0

Table 2.4.3: Column reduced cost matrix

Companies	Projects						
	ED	EN	PH	MA	CE	HS	OL
SA	31	39	145	105	16	141	0
JA	134	70	133	88	15	183	0
ME	23	85	197	78	47	114	0
VL	77	79	109	108	19	141	0
HG	218	43	0	0	0	41	4
KA	4	0	111	0	17	14	0
FJ	0	6	59	5	51	0	0

The solution is not optimal hence, we cannot make feasible assignment.

Table 2.4.4: Column reduced costs matrix

Companies	Projects						
	ED	EN	PH	MA	CE	HS	OL
SA	16	24	130	90	1	126	0
JA	119	55	118	73	0	168	0
ME	8	70	182	63	32	99	0
VL	62	64	94	93	4	126	0
HG	218	43	0	0	0	41	19
KA	4	0	111	0	17	14	15
FJ	0	6	59	5	51	0	15

The above solution is not optimal and so we cannot make feasible assignment.

The solution in the table above is not optimal and hence we cannot make feasible assignment.

The solution in the above Table is optimal. Hence, we make feasible assignment.

SA → EN

JA → CE

ME → ED

VL → OL

HG → PH

KA → MA

FJ → HS

The total cost that will minimize the cost of executing the projects is given as:

$$\text{Total} = 207 + 213 + 118 + 0 + 105 + 140 + 98$$

Total = 881 (in millions of Naira)

Table 2.4.5: Column reduced costs matrix

Companies	Projects						
	ED	EN	PH	MA	CE	HS	OL
SA	8	16	122	82	1	118	0
JA	111	47	110	65	0	160	0
ME	0	62	174	55	32	91	0
VL	54	56	86	85	4	118	0
HG	218	43	0	0	8	41	27
KA	4	0	111	0	25	14	23
FJ	0	6	59	5	59	0	23

Table 2.4.6: Column reduced costs matrix

Companies	Projects						
	ED	EN	PH	MA	CE	HS	OL
SA	8	0	106	66	1	102	0
JA	111	31	94	49	0	144	0
ME	0	46	158	39	32	75	0
VL	70	40	70	69	4	102	0
HG	234	43	0	0	24	41	43
KA	20	0	111	0	41	14	39
FJ	16	6	59	5	75	0	39

2.5 Comparison of the Hungarian method to the Alternate method of solving assignment problem

Having considered the two methods in solving the project tender problem, examples were sourced (see appendix A) and solved using the two methods. The results obtained showed that the Alternative method of assignment gave the solution in less number of iteration. The table below shows the results obtained and the number of iterations used in the three assignment problems considered.

Table 2.5: Comparison table

S/N	Hungarian method		Alternative Method of Assignment	
	No. of iteration	Result	No. of iteration	Result
1	5	881	3	881
2	3	50	2	50
3	3	147	2	147

3. Conclusion

This research seeks to compare and criticize the Hungarian method to the Alternate method of Assignment. After analysis of data, it was observed that the total minimum cost of executing the six projects in both methods (i.e. Alternate method of Assignment and the Hungarian method) was 881 (in millions of Naira). Hence, the companies are assigned as follows

- Sam & Vic Ltd \longrightarrow Engineering block
- Jipdant Association Ltd \longrightarrow Central lecture hall
- Minno Engineering Ltd \longrightarrow Education block
- Hytect Global Ltd \longrightarrow Pharmacy building
- Kafenal Ltd \longrightarrow Management science building
- Fine Job Ltd \longrightarrow Health science building.

It was also realized that the Alternate method of assignment yields optimal solution in fewer steps than the Hungarian method thereby saving time. The computational time it will take to solve this problem is not considered in this work and will make an interesting area for future research.

4. Recommendations

The use of the Alternate of solving assignment problem by [7] gives a systematic and transparent solution as compared to the Hungarian method. Using the more scientific assignment problem model for the placement and selection of the companies to the various projects gives a better result. Management may benefit from the proposed approach for placement and selection of companies to guarantee optimal results from companies. The researcher, therefore, recommends that the Alternate method of solving assignment problem should be adopted by the institution and other institutions and agencies for tender/contractor placement and selection planning. It is also recommended that further research should be done in this area by other researchers.

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APPENDIX A

Example 1

In a textile sale emporium, 4 sales girls Arpitha (A_1), Archena (A_2), Aradhana (A_3) and Aakanasha (A_4) are available to 4 counters M, N, O and P. Each sales girl can handle any given counter. The service of each sales counter (in hours) when carried out by each sales girl is given below

Sales Girl		A_1	A_2	A_3	A_4
Sales counter	M	41	72	39	52
	N	22	29	49	65
	O	27	39	60	51
	P	45	50	48	52

Source: (Page 144 Operations Research, Directorate of Distance Education, Lovely Professional University). www.ipude.in

Example 2: A company has 4 machines to do 3 jobs. Each job can be assigned to one and only one machine. The cost of each job on a machine is given in the following table.

Machines		W	X	Y	Z
Jobs	A	18	24	28	32
	B	8	13	17	18
	C	10	15	19	22

What are the job assignment which will minimize the cost?

Source: Operations Research by Sharma, J. K. 5th edition Page 327, question 3