International Journal of Sciences: Basic and Applied Research (IJSBAR)

ISSN 2307-4531
(Print & Online)

http://gssrr.org/index.php?journal=JournalOfBasicAndApplied

---

# Representation of Frequency and Time Information by Using Wavelets Transform; the Method and Applications

Ali Naji Shaker*

*Directorate of Scholarships and Cultural Relations, Ministry of Higher Education and Scientific Research of Iraq, Phone No. :009647814248092*
*Email: alinaji@scrdiraq.gov.iq*

**Abstract**

The Fourier Transform (FT) is the well-known classical representation of signals components by providing the frequency analysis representations of the signals. The Fourier transformation is found with some determinant such as signal dependent transforming, in another word, the FT is helpful with only particular types of signals such as the pseudo-stationary signals and stationary signals [15], whereas the FT is not fulfilling the expectations while it's being used with non-periodic signals such as noise, and non-stationary signals. As an alternative technique, a Wavelets Transformation (WT) was proposed to perform the frequency analysis for such kind of signals. Since it's a revolved theory and is not broadly famous as compared with FT and other techniques. In this paper, we are going to review the wavelets theory with analysis and demonstrate the applications of this technique.

***Keywords:*** Wavelets Transformation; Fourier Transform; short time Fourier transformation; continues wavelets transform; Partial differential equations; Ordinary Differential Equations.

## 1. Introduction

In order to get a way for Wavelets transforming it is important to understand the Fourier transformation, the easiest manner to shift from the Fourier transformation to the Wavelets transformation is a short time Fourier transformation STFT. The STFT is done by applying the Fourier transform to a selected part of the signal (window) and the shifting that window alone the signal [4].

------------------------------------------------------------------------

* Corresponding author.

The wavelets transforming can be done in either of; continues wavelets transform method, the true discrete transformation of wavelets and continues discredited method. Recently many fields have adopted the wavelets transform for analysing their problems such as engineering applications, science and the technology. In this paper, we are going to analysis a known signal by using different analysis techniques. A signal composed of three components; sin wave with F1 frequency starting at a T1 time, the pulse signal with an F2 frequency at a T2 time frame that is changing again into sin wave at T3 with F3 frequency and finally sin wave at the F4 frequency at a T4 time frame. The signal is shown below and the frequencies are denoted on the graph.

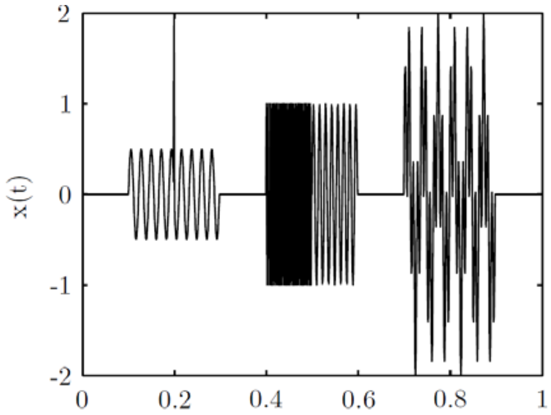


**Figure 1:** test signal comprising of three components to be analysis.

The following table contains the frequencies that are dominated by the signal components at the mentioned time frames:

**Table 1:** The signals' component information

| **Signal** | **Time/sec** | **Frequency/Hz** |
|---|---|---|
| sinusoidal | 0.1-0.3, pulse at 0.2 | 45 |
| sinusoidal | 0.4 | 250 |
| sinusoidal | 0.5 | 75 |
| Sinusoidal *2 | 0.7-0.9 | 30 and 110<br><br>For both signals respectively |

The signal will get sampling in 1000 Hz. In order to switch into wavelet transformation, it is necessary to get way with the Fourier transformation, the short time Fourier transformation will be discussed in the following section.

## 2. Fourier/STFT Analysis

This can be considered as the most widely used method for representation the signals by their frequency components. It is invented by Joseph Fourier at French by 1807. FTT is found with some limitations such as; it can only provide the frequency (Global) representation of the signals' participants. As an alternative, a new approach has been invited to provide the time representation as a top off to the global frequency representation, the approach is well known as short-time Fourier transformation (STFT) [1, 2]. The process of STFT can be performing by selecting a limited window from the signal and then computing the Fourier transform of this window, the margins than are moving along the signal time axis. This window can be denoted as $k(t)$, "e" is the intermediate point of the limited window, the signal to be investigated is denoted by $g(t)$. The global formula of Fourier transform is represented by:

$$h(f) = \int_{-\infty}^{\infty} g(t)e^{-2\pi f t} \qquad (1)$$

The short time Fourier transform can be calculated by multiplying the windowed part of the signal "by" the classical Fourier transformation of the entire signal [3].

If the signal is $g(t)$ and the selected window is $k(t)$ which is centered at "e", hence the $G(e, f)$ is the STFT of the signal. [4] The formula can be rewritten as follows:

$$G(e, f) = \int_{-\infty}^{\infty} g(t) * k(t-e)\, e^{-2\pi f t}\, dt \qquad (2)$$

Where the $k(t-e)$ is the conjugating complex part of the window. The outcomes from this transformation are varied in their efficiency depending on the size of the window. For better resolution of the time, a small window could be chosen. In another hand, a good frequency resolution can be yielded from bigger window outcome. It seems not possible to achieve both time and frequency good resolution at once, such inequalities are termed as Heisenberg. [5] This concept can be illustrated in the following graphs: figure 2a is presenting the time resolution achieved by a small window; it is cleared that window has only deployed 0.05 second from the time axis and figure 2b is showing the frequency resolution with bigger (longer) window. It is also cleared that best frequency results were gained by 0.59s deployment length on the time axis in the figure 2b. Both figures are demonstrating the time and frequency representations of the signal that was undergoing the Short Time Fourier transformation (STFT).

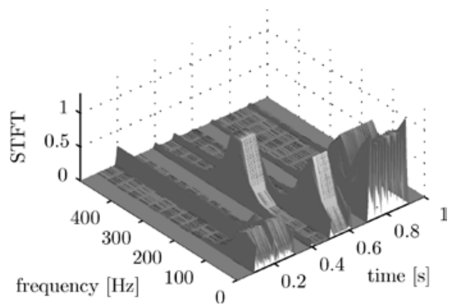


**Figure 2a:** Best resolution of time

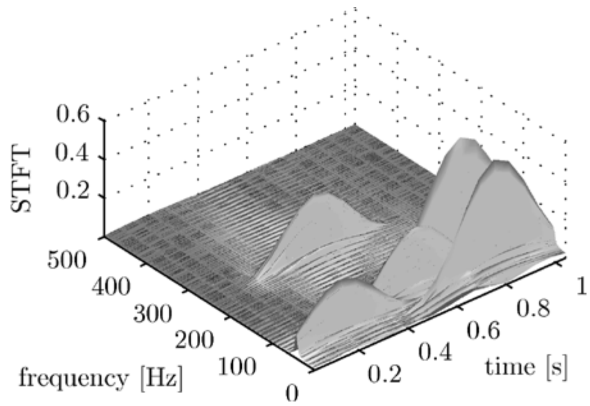


**Figure 2b:** Best resolution of frequency

At this point, it is necessary to find the way in which we can achieve an acceptable time and frequency resolutions. Since those values are looking critical then the only way get acceptable outcomes is done by adjusting the window size [5].

For this example, a window of 0.16 seconds length may be chosen in order to get moderated results in both time and frequency resolutions.

The figure below (2c) is showing the final size of the window that yields the results stated above.

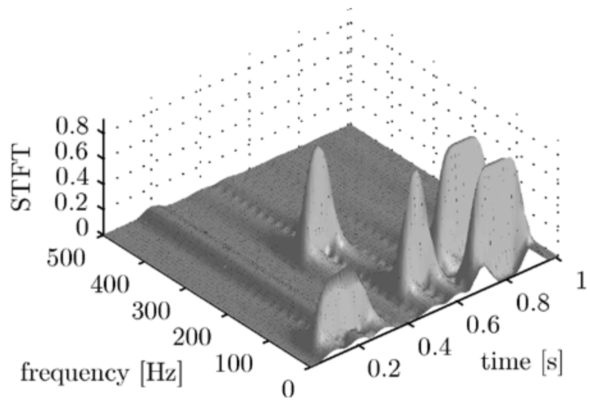


**Figure 2c:** Final window adjustment

**3. Analysing by Wavelet**:

Since the other analysis methods such as Fourier/STFT are not performing well when it was about to mobilizing signals, a wavelet is considered as the best alternative. [6] It can compute the signal space as well as the multi-resolution analysis with a more freedom and less compromising in critical values of time and frequency resolutions. To overcome these issues a wavelet analysis has been proposed, it is achievable by computing the mother wavelet/the wavelet function $\Psi(t)$ and henceforth the convolution between this function and the signal of interest must be calculated in order to perform the wavelet analysis. The mother wavelet $\Psi(t)$ can be chosen freely as compared to the procedure of windowed signal election in the STFT, it has to take a unique frequency oscillation to be more recognized and segregated from the other frequency components in the signal. [4] The following figure is demonstrating an example of the wavelet called as Morlet.

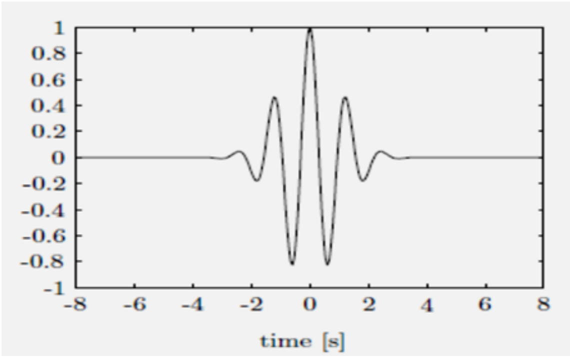


**Figure 3:** An example of Wavelet signal

The wavelet mother may act as a wavelet if the energy terms of it is not infinite such as:

$$Energy(t) = \int_{-\infty}^{\infty} |\Psi(t)|^2 \quad \Delta t$$

where "t" (time representation) must be lesser than infinity. Another criterion such as the zero frequency sections of the wavelet does not exist must be assured as it explained in the following formula:

$$\mathrm{Z}_{\Psi} = \int_{0}^{\infty} \frac{|\,\Psi(\mathrm{f})|^2}{\mathrm{f}} \quad \Delta\mathrm{f}$$

The result needs to be lesser than infinity as well. And finally, the wavelet transform must be same to the non-positive frequencies.

**4. CWF analysis**

For the input signal $m(t)$, the continuous wavelet transformation can be evaluated by applying the convolution between the signal of interest $m(t)$ and the wavelet mother/analysis function $\Psi(\mathrm{t})$. Two parameters were monitored in the following formula; the scaling index "c" and the parameter of translating "$\gamma$" [7].

$$M(\gamma, c) = (c^{-0.5}) \ . \int_{-\infty}^{\infty} m(t). \Psi^{*}(\frac{t-\gamma}{c}) \quad \Delta t$$

The final mathematical interpretation of this convolution is given above by M $(\gamma, c)$, where c is representing the absolute value of this parameter.

The importance of the scale parameter "c" lies in changing the window length and the Centre frequency of the window i.e. $f_c$. The scale parameter "c" is employed to play the role of wavelet analysis representing instead of the use of frequencies. [8] On another hand, the importance of translating parameter $\gamma$ is seemed to move the window along the axis of time as the $\gamma$ value is dominating the intermediate point of the window so that the location at the time axis can be determined accordingly.

The participants of the above formula are known as coefficients of the wavelet, each one of it is being associated with time and frequency points lying on the axis. By applying the concept of inverse wavelet transformation, [4] the original signal can be revealed. That can be yielded from the following formula:

$$m(t) = C_{\Psi}^{-2} \,.\, \iint_{-\infty}^{\infty} M(\gamma, c) * c^{-2} * \Psi\left(\frac{t-\gamma}{s}\right) \Delta c \, \Delta\gamma$$

The admissible constant is referred as "$C_{\Psi}$" and it is mandatory for satisfying the wavelet conditions stated in the previous section. The mother wavelet is centered at fc/ the intermediate frequency of the wavelet function. The scale parameter (c) is inversely proportional to the intermediate frequency fc.

The issue of constant inequalities (Heisenberg) still exists in the wavelet analysis, in other word by reducing the scale parameter "c" a smaller window can be produced yielding a good time resolution with lesser frequency resolution. It can be said that wavelet has a relatively fixed resolution of frequency. The intermediate frequency fc and the scaling parameter c can be combined in one mathematical representation which is given in the following relation: $F=F_{centered}/c$.

For the Morlet signal, firstly the window can be derived from the signal as a Gaussian format having the above values of scaling and intermediate frequencies so that following expressions can be written [9].

The mother wavelet is given in the formula as:

$$\Psi(t) = m(t) * \, e^{-2\pi * fcent} * t$$

Where:

$$m(t) = \sqrt{\pi f w} \, * \, e^{t^2 f w^{-1}}$$

In the case of continuous wavelet transformation, the value of the wavelet parameters (time and scaling) are taken in discrete values. [10] That is yielding a series of wavelet signal and in order to convert the above parameters into a discrete value, the discretization process is required.

**5. Discretization method**

The process those look after switching the continuous value into discrete values. It can be considered as most powerful and effective method of discretization. It is well known as the dyadic grid. The last is used to produce the Q results. The property of Q is the factor of quality that is used to present the frequency fixed resolution of the wavelet analysis [4]. The series of wavelet can be achieved by the following expression.

$$Y\ wavelet = \int_{-\infty}^{\infty} m(t)\ \Psi_{b,r}(t)\ \ \Delta t$$

Where:

$$\Psi_{b,r}= C_0^{-05b}\ \Psi(C_0^{-b}\ t-r\gamma_0)$$

The translation and dilation of the wavelet are controlled by the (r and b) parameters respectively. For $\gamma_0\ and\ C_0$ these values are equal to zero in a case of the dyadic grid.

For the signal example illustrated in figure 1, the discrete wavelet analysis results are given the following figures. The figure below is showing best results in the resolution of frequency within shrink time scaling. [11] And best time resolution within the shorter frequency scale. The scales are explained in previous sections and stated to be controlling the window for best results in frequency and time scales.

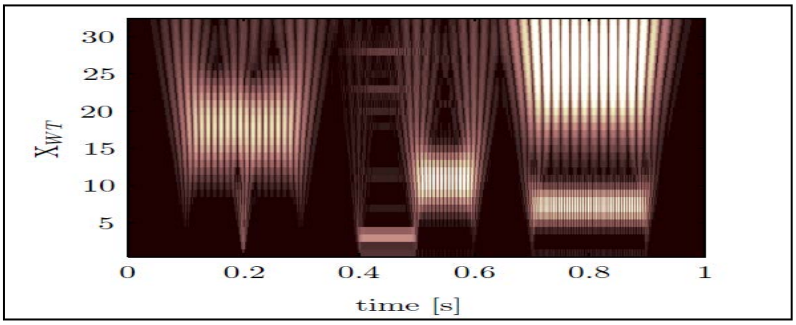


**Figure 4:** plot of the contour

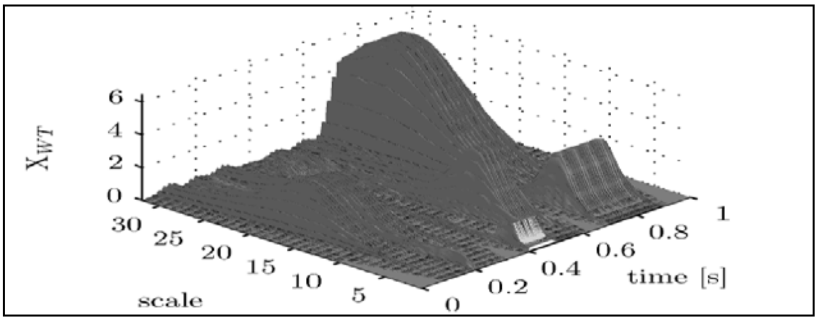


**Figure 5:** plot of the surface

## 6. Applications

In this section, we are going to discuss the some of the applications which are using the wavelet theorem for analysing their problems. The wavelet application is ranging from the engineering and science with other fields also such as medical and finance. At this point, we are interested in highlighting the mathematical applications hence; the following sections are involving the significance of wavelet analysis in the numerical systems.

### *6.1 Differential Equations (Ordinary)*

The DQs can be solved with help of wavelet transformation the solution can be executed by taking the Fourier transform to the right and side of the equation and then dividing by the coefficients of wavelet series [14]. The DQ may take the following format to be solved with the wavelet analysis.

$$Mv(x) = f(x)$$

$$when \quad x \, \epsilon \, [1, 0] \quad and \quad M = \sum_{a=0}^{n} S_a(x) D^a$$

### *6.2 Partial differential equations*

The physical problems are being widely analysing by the partial differential equations (PDQ). Practically, it's a complex task to compute the analytical solution of the PDQ, as a result to this; it seems to be important to employ some numerical method in order to achieve the solution. A wavelet is an efficient tool for solving the differential equations by applying to track the slope position to improve the resolution of the local grid by adding more resolution wavelet [12].

### *6.3 Image processing*

As a time-frequency scale transformation tool for data, function and operator, the wavelet transform is a very good method for image compression, by which redundancies of the image are removed and original features of the image are reserved. The pixels of facial images are usually larger, so the wavelet transform is used before image comparison. The low-frequency images can be decomposed and extracted by wavelet transform. The image comparison between the sample images and the testing images, which is based on low-frequency components in different decomposed layer, can reduce effectively the computational complexity and make face recognition very fast. For face recognition, the problem of image comparison can be transformed into sequence comparison. The image comparison offers a similarity degree for human faces after the sequence similarity is defined, using a score function and a comparison function. Finally, a threshold is setting to guarantee the authenticity of correct recognition for human facial images to a certain extent. For the sake of improving computing efficiency, it is necessary to normalize different values of similarity degree. The experiments show that the wavelet method has the characteristics of simple realization, rapid recognition speed and high recognition rate. [5,13]

## 7. Conclusion

The wavelet approach is reviewed in this paper by showing the significance of this method to perform the analysis of frequency domain presentation of time domain signals aiming to investigate the frequency analysis and time analysis, unlike the Fourier transformation the wavelet transformation is yielding both representations of frequency and time components of the signal. The main efficient difference achieved from the wavelet analysis is the freedom of choosing the window over the time domain by performing the convolution concept. More effective frequency and time resolutions in the results can be gained from this method. The applications of wavelet transformation are huge due to the ability to perform the analytical solutions with easier fashion by using the numerical methods those employing the wavelet transformation.

**References**

[1] B. de Kraker. A numerical-experimental approach in structural dynamics. Technical report, Eindhoven University of Technology, Department of Mechanical Engineering, 2000.

[2] J.J. Kok and M.J.G. van de Molengraft. Signal analysis. Technical report, Eindhoven University of Technology, Department of Mechanical Engineering, 2002.

[3] J.F. James. A student's guide to Fourier transforms. Cambridge University Press, first edition, 1995. ISBN 0-521-46829-9.

[4] M.G.E. Schneiders. Wavelets in control engineering. Master's thesis, Eindhoven University of Technology, August 2001. DCT nr. 2001.38.

[5] O. Rioul and M. Vetterli. Wavelets and signal processing. IEEE SP Magazine, pages 14–38, October 1991.

[6] M. Misiti, Y. Misiti, G. Oppenheim, and J-M Poggi. Wavelets Toolbox Users Guide. The MathWorks, 2000. Wavelet Toolbox, for use with MATLAB.

[7] P.S. Addison. The Illustrated Wavelet Transform Handbook. IOP Publishing Ltd, 2002. ISBN 0-7503-0692-0.

[8] R. Polikar. The wavelet tutorial. URL:http://users.rowan.edu/polikar/WAVELETS/WTtutorial.html, March 1999.

[9] J.N. Bradley and C.M. Brislawn. The wavelet/scalar quantization compression standard for digital fingerprint images. IEEE Circuits and Systems, 3:208–208, May 1994.

[10] G. Strang and T. Nguyen. Wavelets and Filter Banks. Wellesley-Cambridge Press, second edition, 1997. ISBN 0-9614088-7-1.

[11] C.M. Chang and T.S. Liu. Application of discrete wavelet transform to repetitive control. Proceedings of the ACC, pages 4560–4565, May 2002.

[12] P. Cruz, A. Mendes, and F.D. Magalh aes. Using wavelets for solving PDEs: and adaptive collocation

method. Chemical Engineering Science, 56:3305–3309, 2001.

[13] B.E. Usevitch. A tutorial on modern lossy wavelet image compression: Foundations of JPEG 2000. IEEE Signal Processing Magazine, pages 22–35, September 2001.

[14] W. Sweldens. Construction and Applications of Wavelets in Numerical Analysis. Ph.D. thesis, Department of Computer Science, Catholic University of Leuven, Belgium, May 1995.

[15] R. A. De Vore, B. Jawerth, and B. J. Lucier, Image compression through wavelet transform coding, preprint.